Engineering Sciences

Synchronization control of DC motors through adaptive disturbance cancellation

– *Stability analysis* –

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Summary

- Electric motor model
- Control problem
- Stability analysis
- Logic simulation

Electric motor model

$$\dot{\theta}(t) = \omega(t)$$

$$\dot{\omega}(t) = \frac{-F}{J}\omega(t) - c_1 - c_2\theta(t) + \frac{k_M}{J} i(t)$$

Uncertain motor parameters

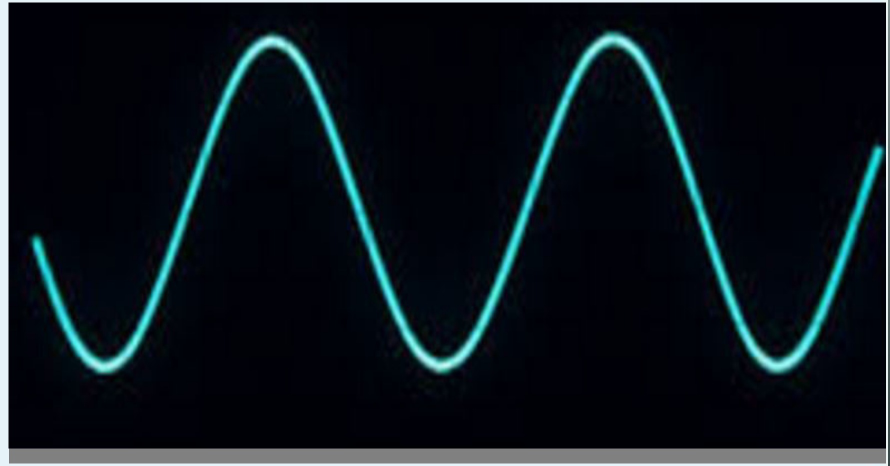
- F → Viscous friction coefficient
- J → Motor inertia
- k_M → Motor torque constant

Load torque approximation → $Jc_1 + Jc_2\theta$

Control problem

Reference sinusoidal signal

$$\theta_r(t) = A_b + A\sin(\gamma t + \overline{\omega})$$



Tracking signal

$$\theta(t)$$

⇨ Reference input current

$$i_r = \frac{J}{k_M}\left[\ddot{\theta}_r + \frac{F}{J}\dot{\theta}_r + c_1 + c_2\theta_r\right] \longrightarrow$$

Same frequency of θ_r

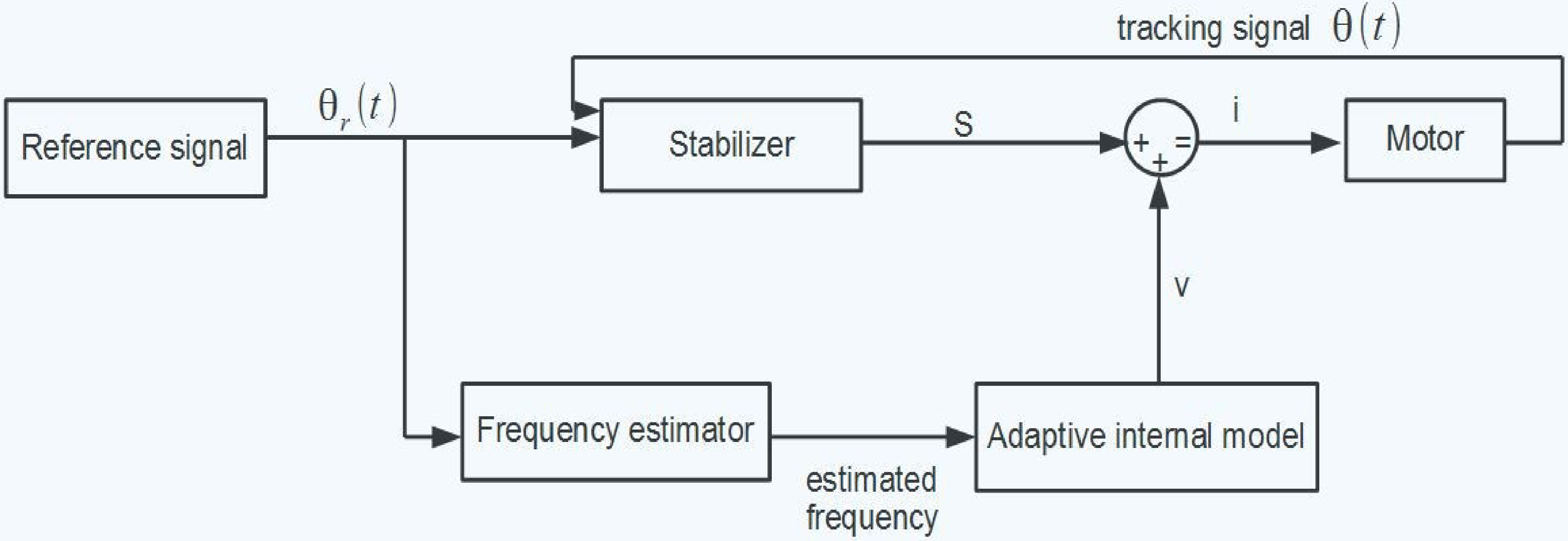
Time domain $\longrightarrow t$

$$i = -2\lambda k_y \tilde{y}(t) - k_z\left[\xi(t) + k_y \tilde{y}(t)\right] - \hat{w}_0(t) - \hat{w}_1(t)$$

Laplace domain $\longrightarrow s$

$$L\{i\}(s) = -\bar{a}\frac{(s+\lambda)}{(s+\lambda+k_z)}L\{\tilde{y}\}(s) + L\{v\}(s)$$

with $\bar{a} = k_y(2\lambda + k_z)$



⇨ Modular implementation

⇨ Parallel working → Stabilizing action → S

→ Reconstructive and removing action → v

Stability analysis

Recalling the motor model

$$\dot{\theta}(t)=\omega(t)$$

$$\dot{\omega}(t)=\frac{-F}{J}\omega(t)-c_1-c_2\theta(t)+\frac{k_M}{J}i(t)$$

change of coordinates

$$x_1=\theta$$

$$x_2=\omega+\frac{F}{J}\theta$$

Output feedback form

$$\dot{x}_1=x_2-\frac{F}{J}x_1$$

$$\dot{x}_2=-c_1-c_2x_1+\frac{k_M}{J}i$$

$$y=x_1$$

⇨ Linear filter

$$\dot{\xi} = -\lambda \xi + S$$

$$\xi(0) = \xi_0$$

⇨ change of coordinates

$$z = [z_1, z_2]^T = \left[x_1, x_2 - \frac{(k_M \xi)}{J} \right]^T$$

$$y = z_1, \eta = z_2 - \lambda z_1$$

New system

$$\dot{y} = \eta + \left(\lambda - \frac{F}{J} \right) + \xi \frac{k_M}{J}$$

$$\dot{\eta} = -\lambda \eta - c_1 - \left(c_2 + \lambda^2 - \lambda \frac{F}{J} \right) y + \frac{k_M}{J} v$$

The corresponding reference system:

$$\dot{y}_r = \eta_r + \left(\lambda - \frac{F}{J}\right) y_r$$

$$\dot{\eta}_r = -\lambda \eta_r - c_1 - \left(c_2 + \lambda^2 - \lambda \frac{F}{J}\right) y_r + \frac{k_M}{J} i_r$$

recalling

$$y_r = \theta_r,\ \xi_r = 0,\ v_r = i_r$$

using $\left(\tilde{y} = y - y_r = \theta - \theta_r ;\ \tilde{\eta} = \eta - \eta_r\right)$

$$\dot{\tilde{y}} = \tilde{\eta} + \left(\lambda - \frac{F}{J}\right) \tilde{y} + \frac{k_M}{J} \xi$$

$$\dot{\tilde{\eta}} = -\lambda \tilde{\eta} - \left(c_2 + \lambda^2 - \lambda \frac{F}{J}\right) \tilde{y} + \frac{k_M}{J} \left(v - i_r\right)$$

⇨ new error variable

$$z_\xi = \xi + k_y \tilde{y}$$

⇨ stabilizing action

$$S = -2\lambda k_y \tilde{y} - k_z \left(\xi + k_y \tilde{y}\right)$$

⇨ Final error system

$$\dot{\tilde{y}} = +\tilde{\eta} + \left(\lambda - \frac{F}{J} - k_y \frac{k_M}{J}\right)\tilde{y} + \frac{k_M}{J} z_\xi$$

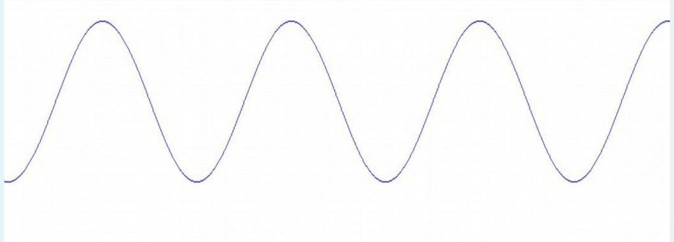
$$\dot{\tilde{\eta}} = -\lambda\tilde{\eta} - \left(c_2 + \lambda^2 - \lambda\frac{F}{J}\right)\tilde{y} + \frac{k_M}{J}\left(v - i_r\right)$$

⇨ Hurwitz

$$\dot{z}_\xi = -\left(k_z + \lambda - \frac{k_M}{J} k_y\right) z_\xi + k_y \tilde{\eta} - \left(\frac{F}{J} k_y + \frac{k_M}{J} k_y^2\right)\tilde{y}$$

⇨ Recalling

$$i_r = \frac{J}{k_M}\left[\ddot{\theta}_r + \frac{F}{J}\dot{\theta}_r + c_1 + c_2\theta_r\right] \rightarrow$$



⇨ Generated by the internal model → In the case of known frequency

$$\dot{w}_0 = 0$$ → Bias

$$\dot{w}_1 = w_2$$
$$\dot{w}_2 = -\gamma^2 w_1$$ → $\pm j\gamma$ eigenvalues → Sine wave

$$-i_r = w_0 + w_1$$

Adaptive internal model

$$\dot{\hat{w}}_0 = k\,\tilde{y}$$
$$\dot{\hat{w}}_1 = \hat{w}_2 + k\,\tilde{y}$$
$$\dot{\hat{w}}_2 = -\hat{\theta}_1\,\hat{w}_1$$

final reconstructive control

$$v = -\hat{w}_0 - \hat{w}_1$$

Frequency estimator

$$\dot{\hat{\theta}}_r = \hat{\eta}_f + \lambda_f\theta_r + \hat{\theta}_1\xi_f\lambda_f + \frac{\hat{\theta}_0}{\lambda_f}k_f(\theta_r - \hat{\theta}_r)$$
$$\dot{\hat{\eta}}_f = -\lambda_f\hat{\eta}_f - \lambda_f^2\theta_r$$
$$\dot{\hat{\theta}}_0 = \lambda_0(\theta_r - \hat{\theta}_r)$$
$$\dot{\hat{\theta}}_1 = \lambda_1\xi_f(\theta_r - \hat{\theta}_r)$$

from

C.M Verrelli, "Synchronization of permament magnet electric motors: non linear advanced results", Non linear Analysis: Real World Applications, 13:395-409, 2012.

⇒ The complete error system

$$\dot{\Pi}_s = \bar{A}_p \Pi_s + B_s(\hat{\theta} - \gamma^2)(w_1 - \tilde{w}_1)$$

⇓

The eigenvalues of the matrix $\bar{A}_p$ characterizing the overall error system coincide with the roots of the polynomial at the numerator of the rational function

$$1 + k\,P(s)\left[\frac{1}{s} + \frac{s}{(s^2+\gamma^2)}\right]$$

from → *R. Marino, P. Tomei, "Output regulation for unknown stable linear systems, IEE Transactions on automatic Control, 60: 2213-2218, 2015*

⇓

Negative real part (k suff. small) ⇒ Convergence → 0

Logic simulation

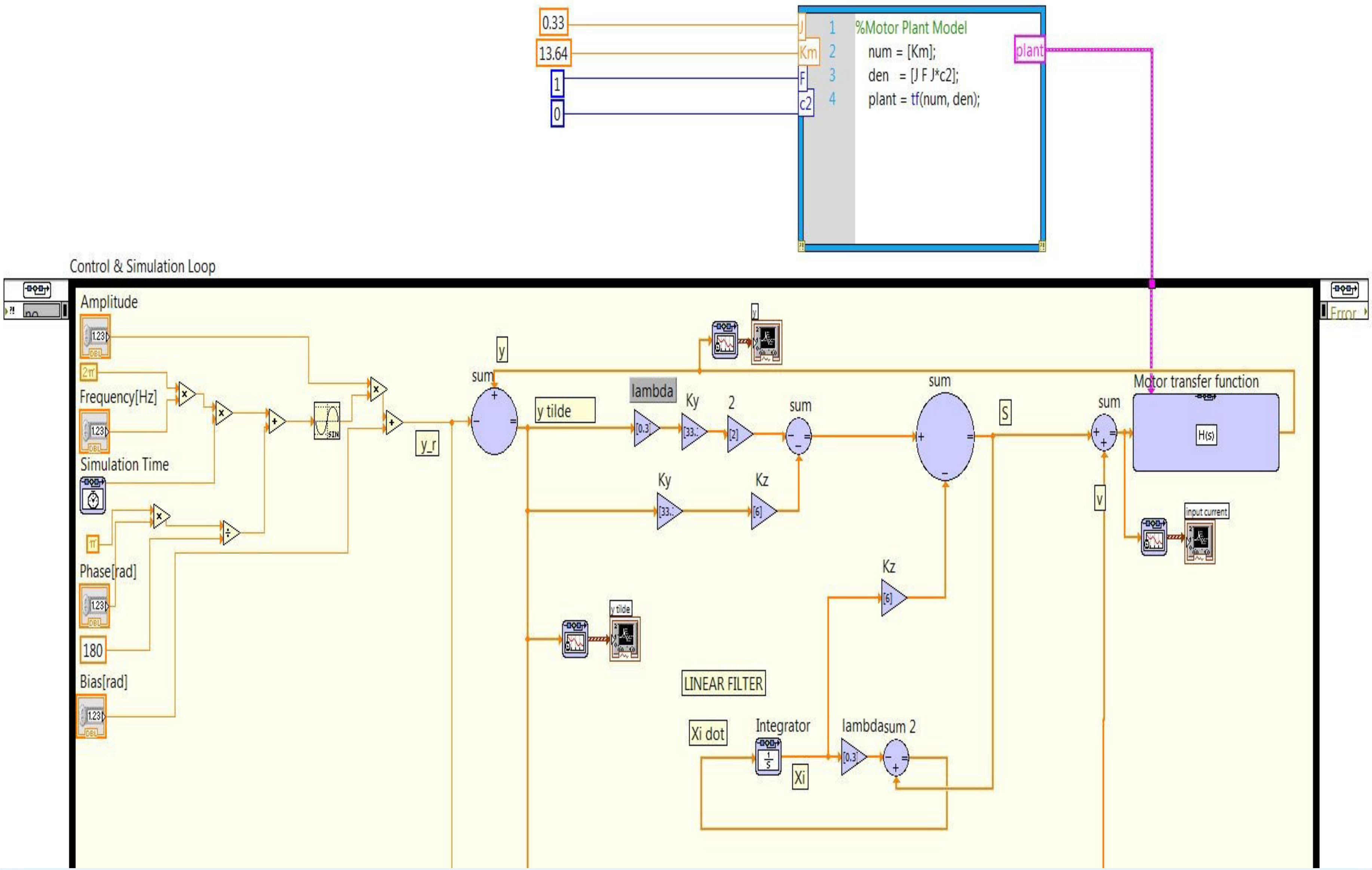
NATIONAL INSTRUMENTS LabVIEW with

Motor parameters

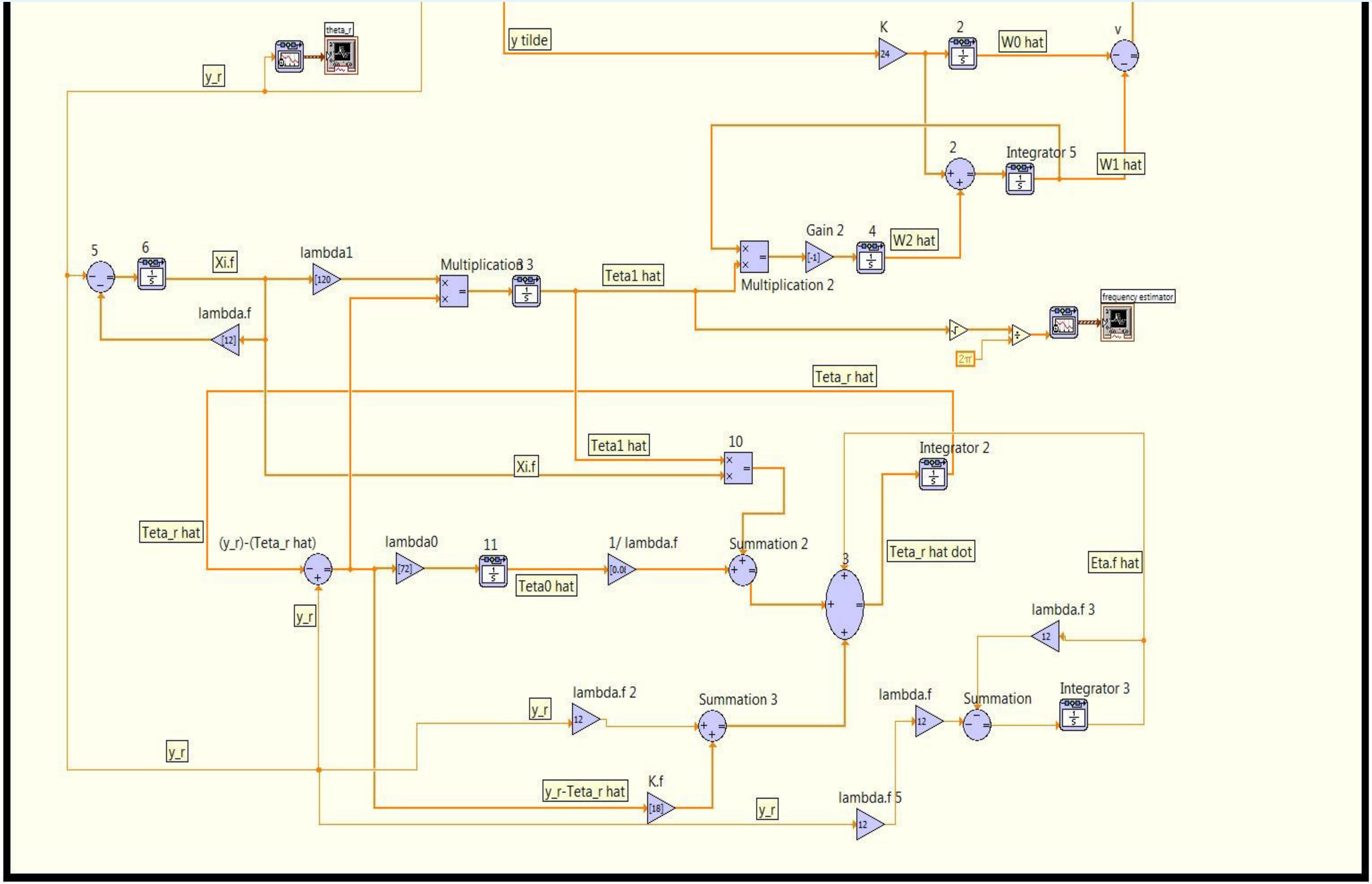
J	k_m	F	c_2
0.33	13,64	1	0

Design parameters

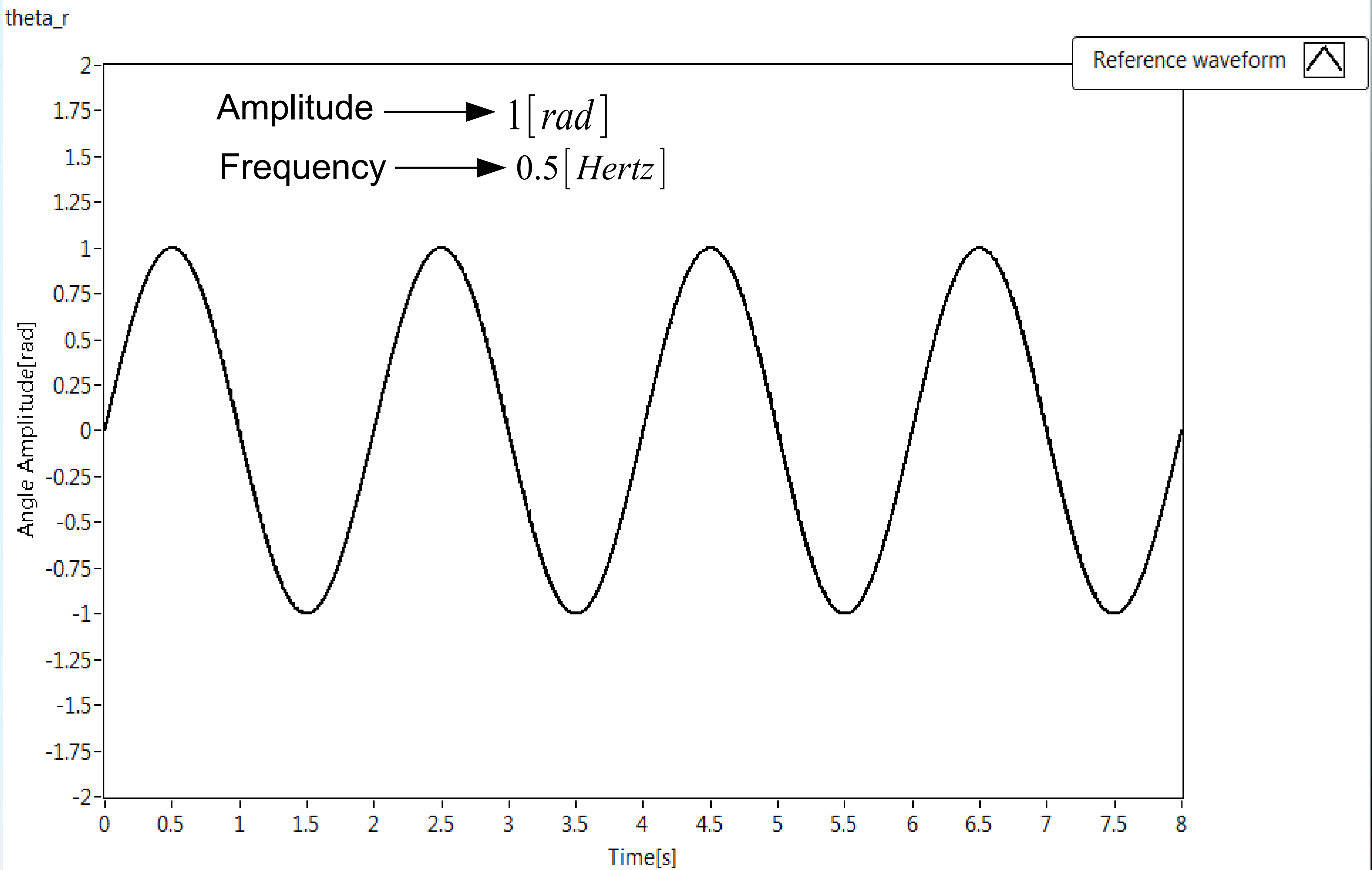
λ	k_y	k_z	λ_1	λ_0	k_f	λ_f
0.3	0.33	6	120	72	18	12



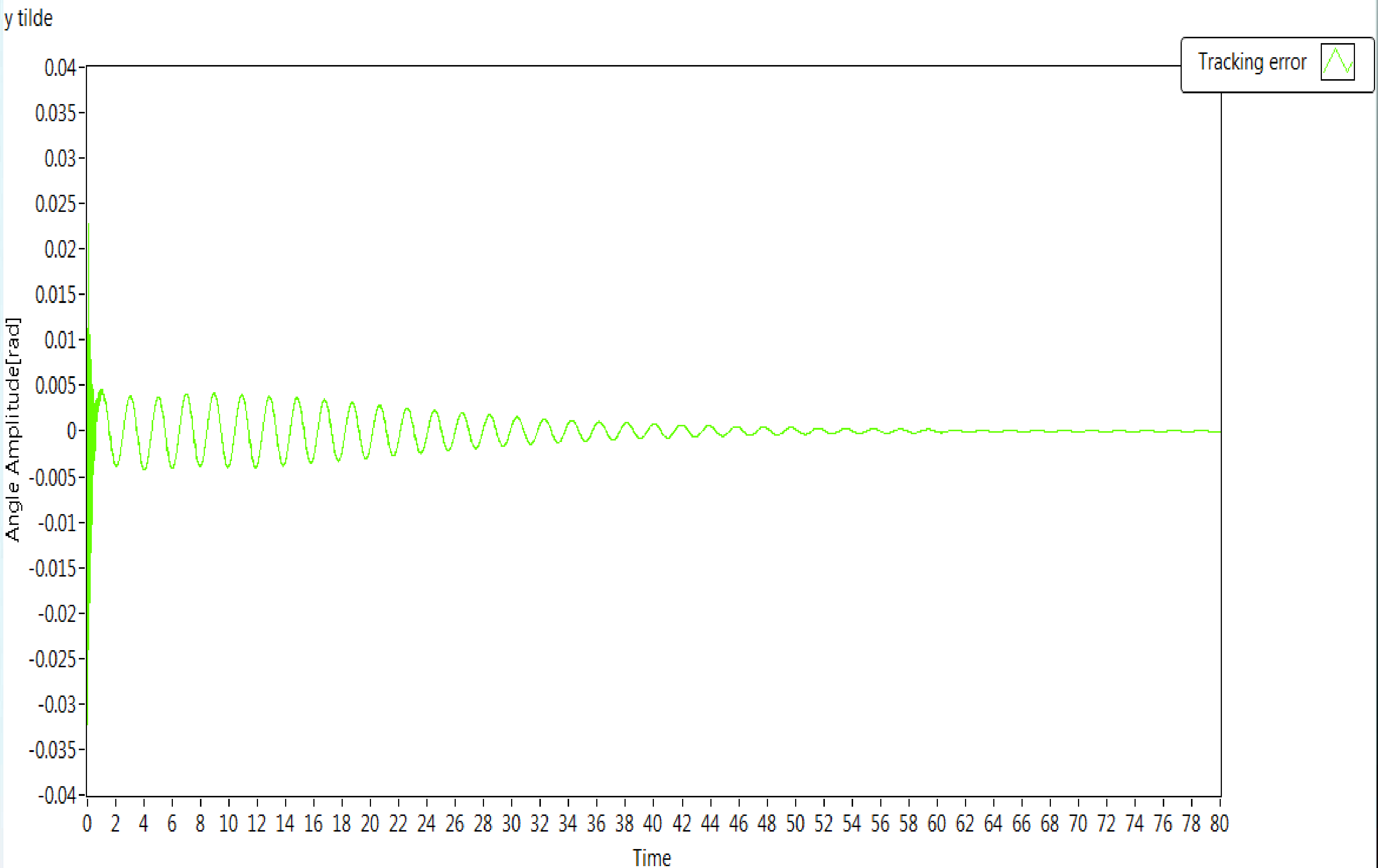
Logic implementation of S



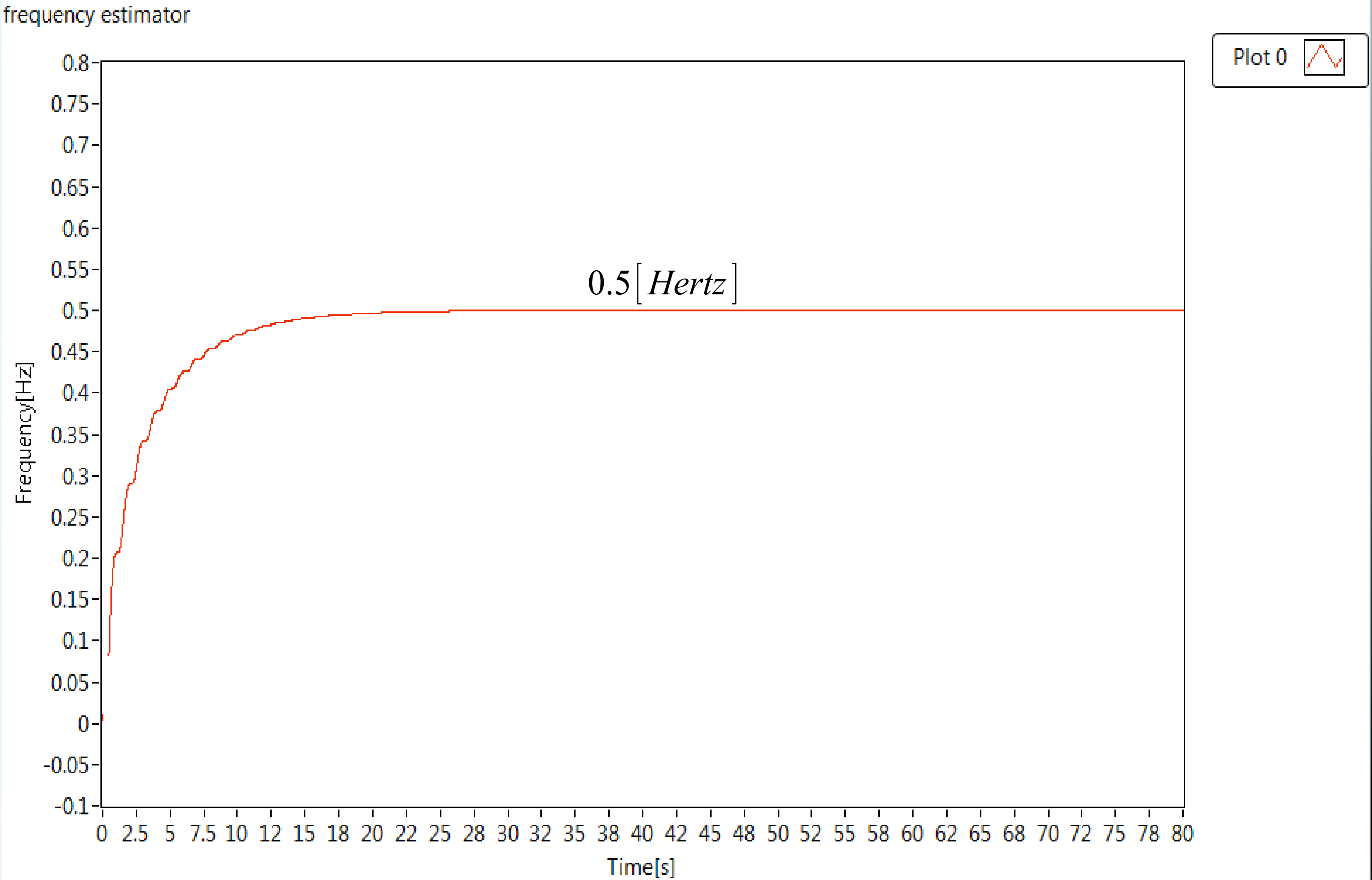
Logic implementation of v



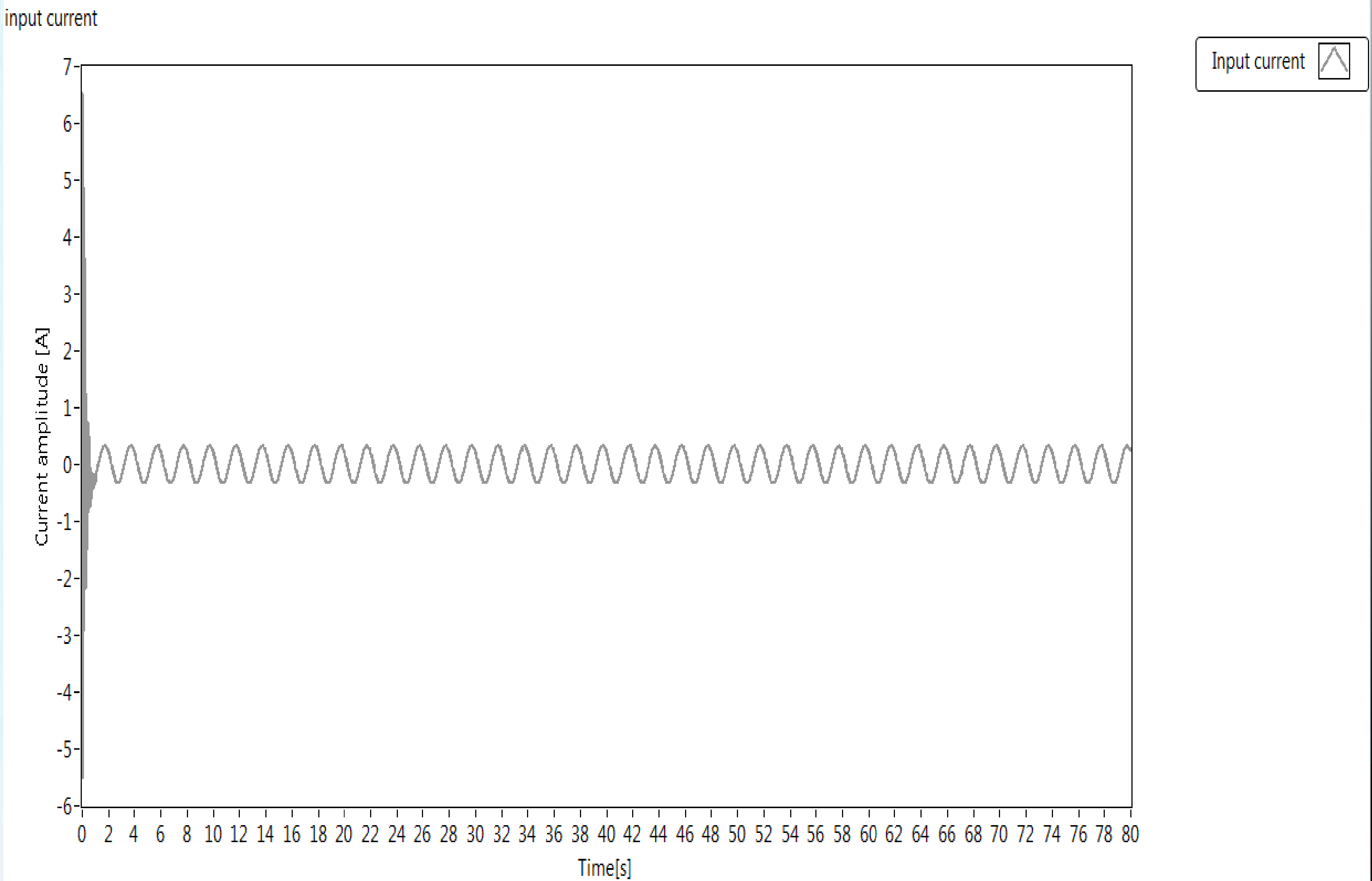
Reference signal



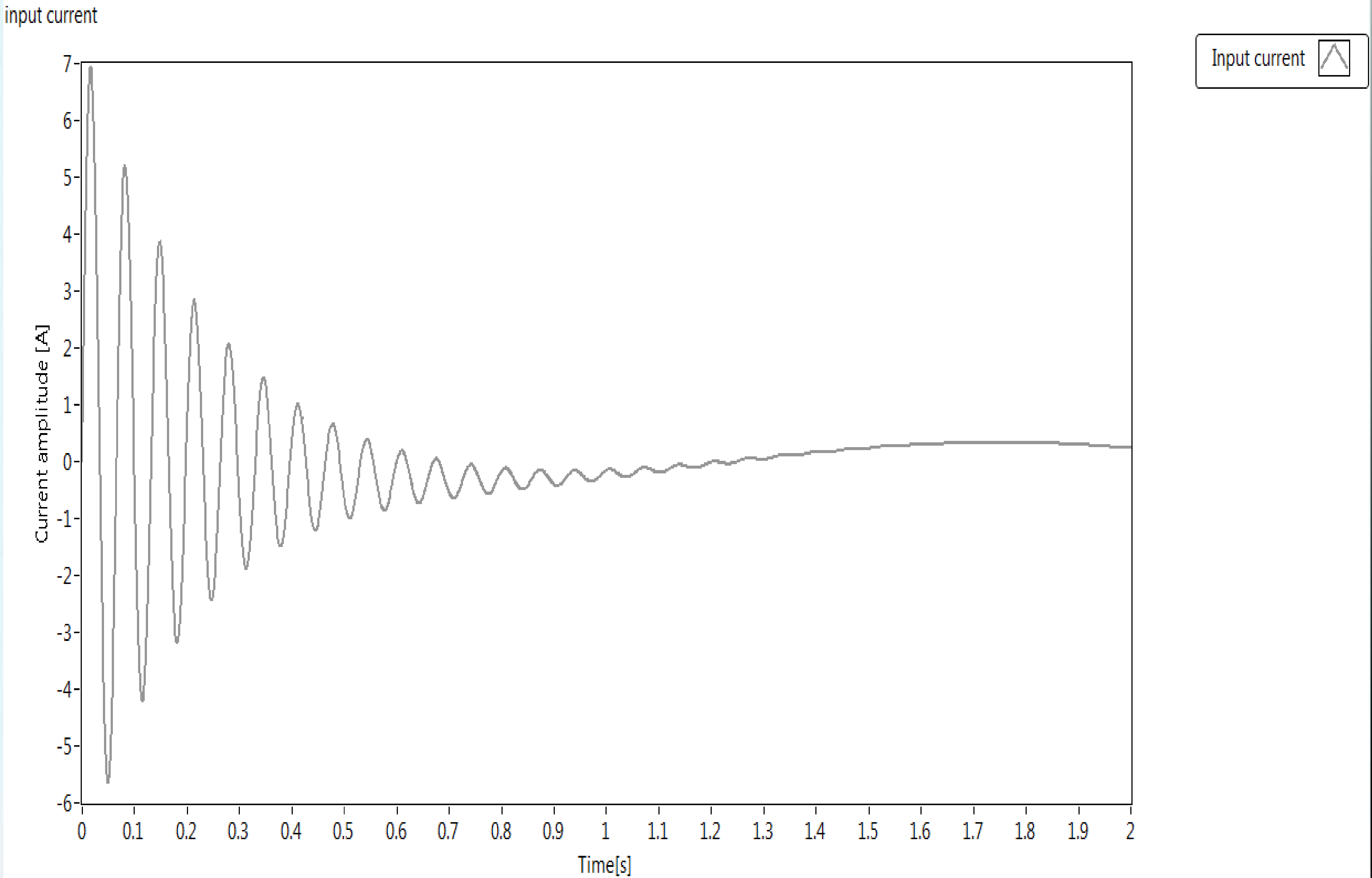
Tracking error



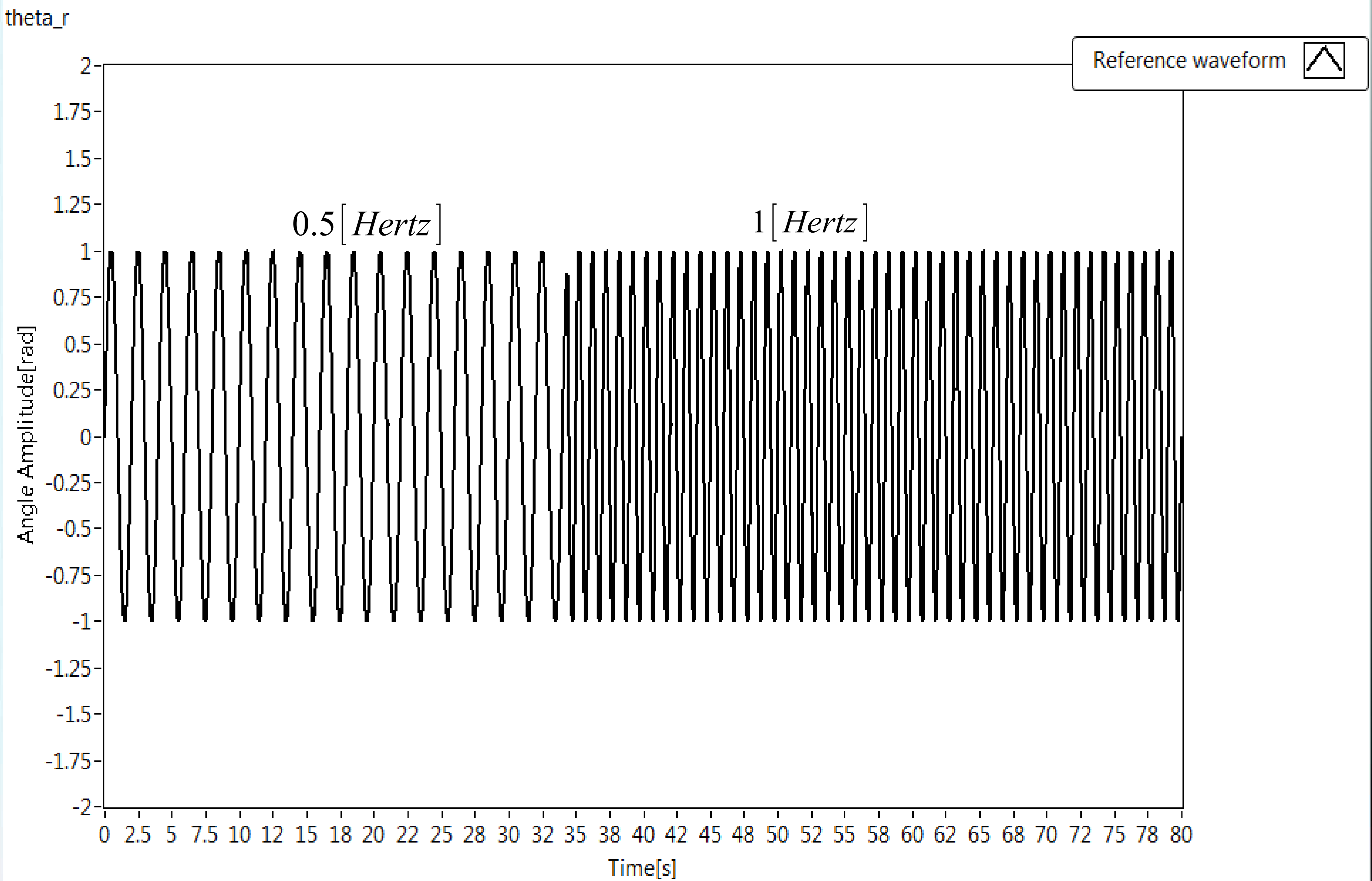
Frequency estimator



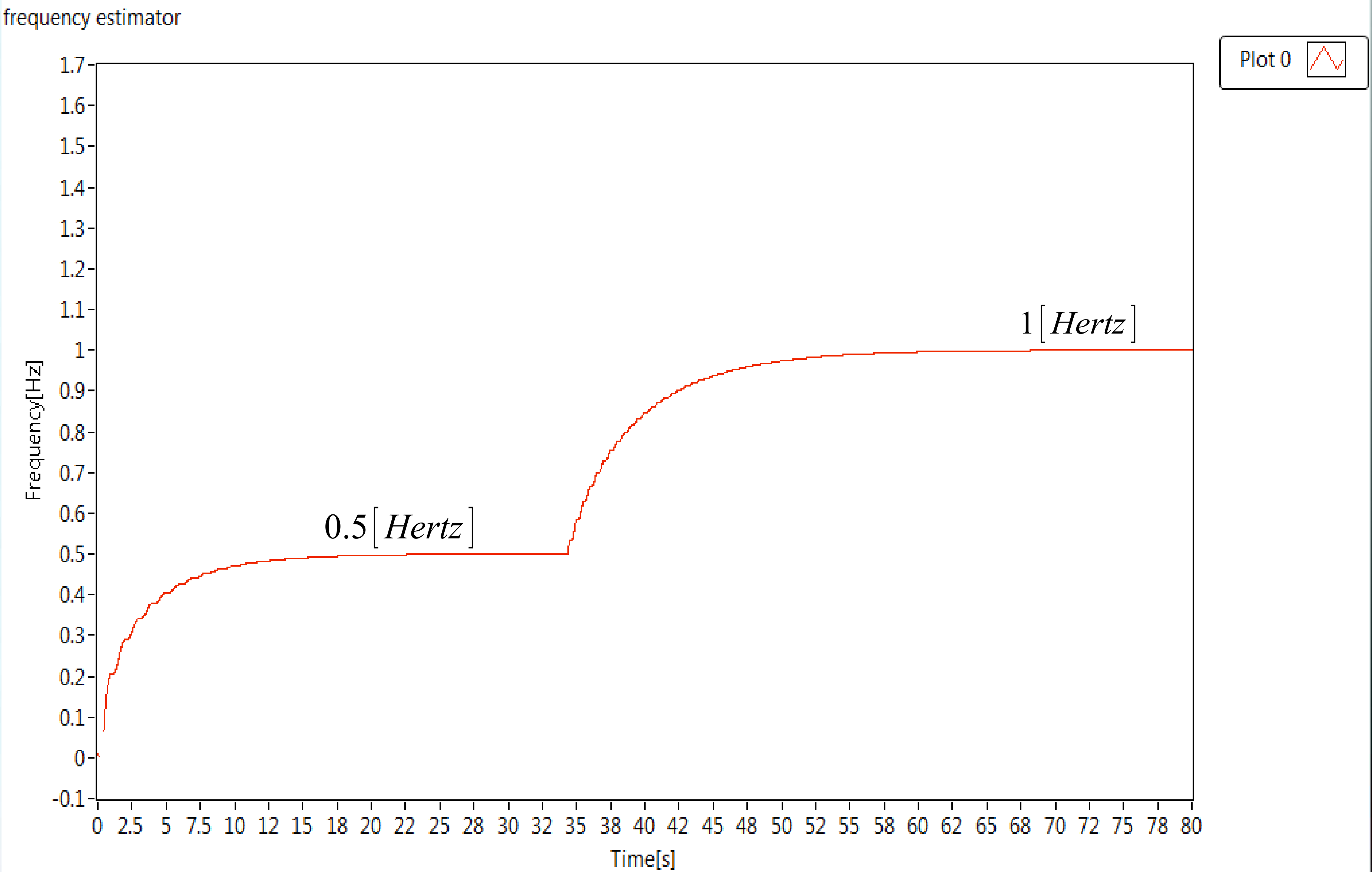
Input current



Input current,transient state



Frequency change



Frequency estimator

Conclusion

- The control is able to track the reference signal using an innovative technique,minimizing the tracking error and to adapt to a change of frequency pretty fast