



UNIVERSITA' degli STUDI di ROMA  
TOR VERGATA

# MULTIPHYSICS SIMULATION BY THE FINITE ELEMENT METHOD

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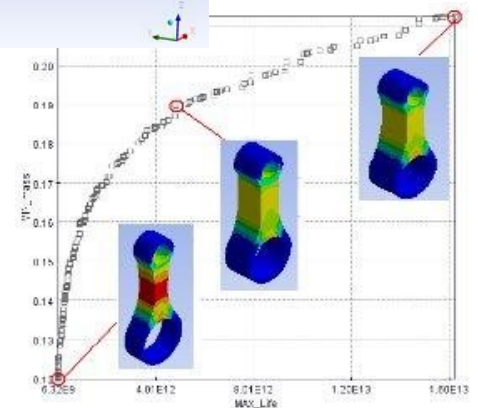
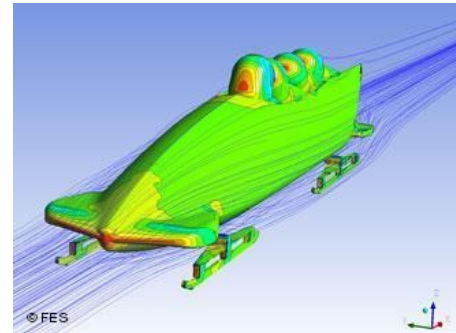
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A.Y. 2013/2014

# Context

**Computer Simulation** has become an essential part of science and engineering for :

- Developing new products
- Optimizing designs
- Describing physical phenomena



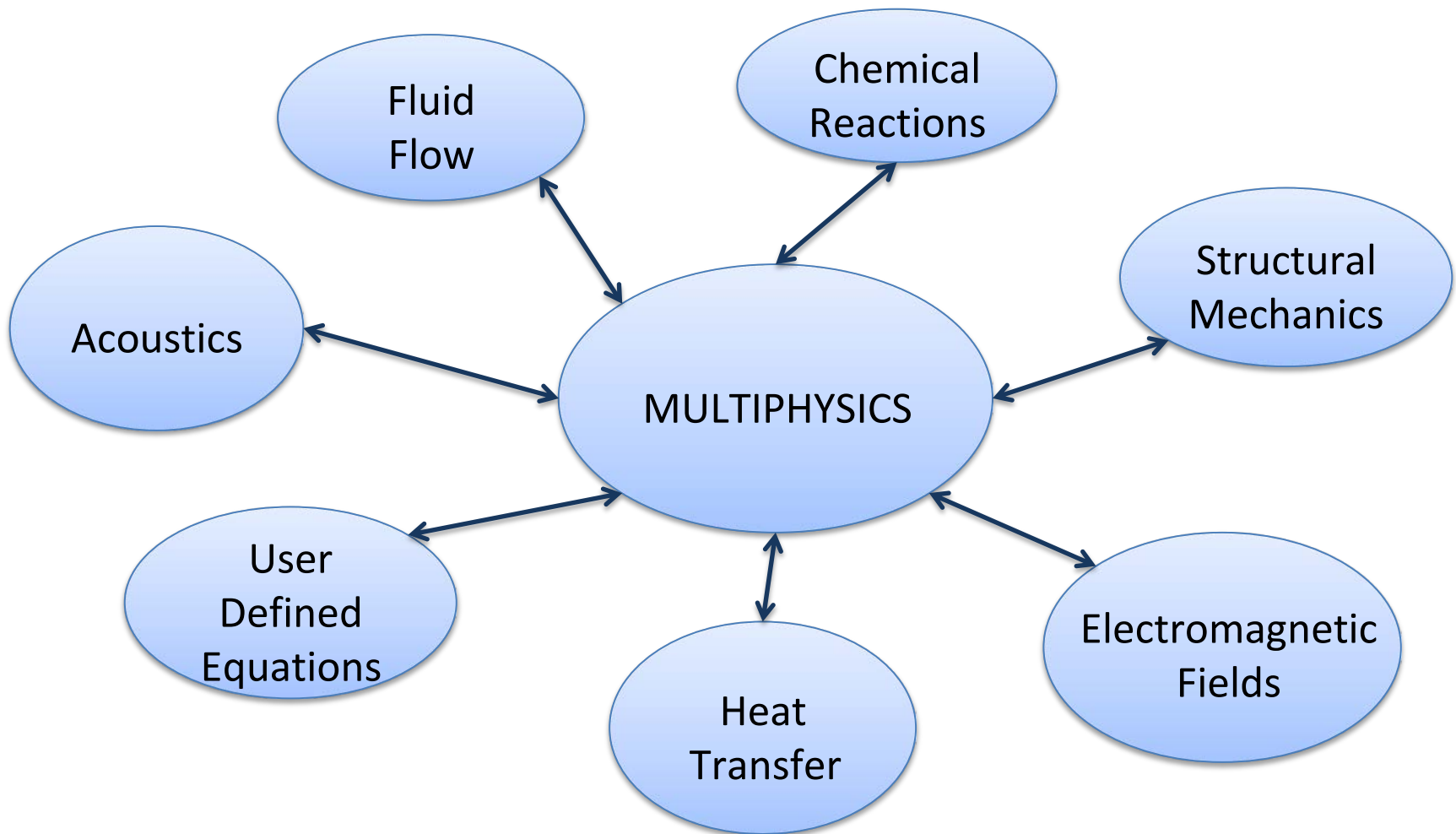
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But a common concern is represented by :

- Modelling **Accuracy**
- Results **Reliability**

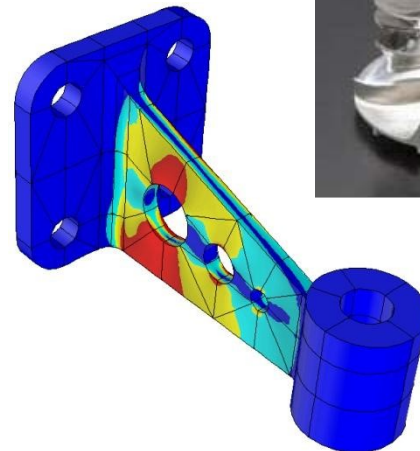
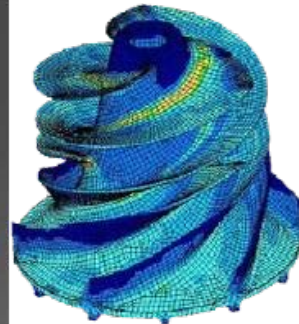
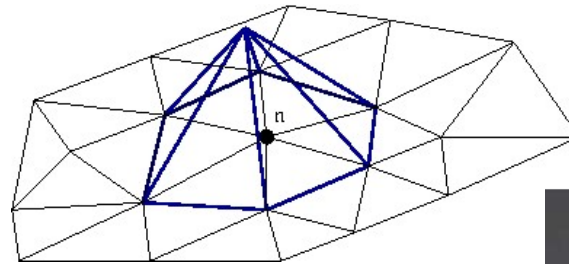
# Multiphysics Simulations

**Multiphysics** treats simulations that involve multiple physical models or multiple simultaneous physical phenomena



# Finite Element Method (FEM)

The **Finite Element Method** (FEM) is a numerical approximation technique that divides a component into discrete regions (the finite elements) where the response of a given physical problem is evaluated.



FEM requires :

- Formulation

- Solution Process
- Material Representation
- Geometry
- Boundary Conditions and Loadings

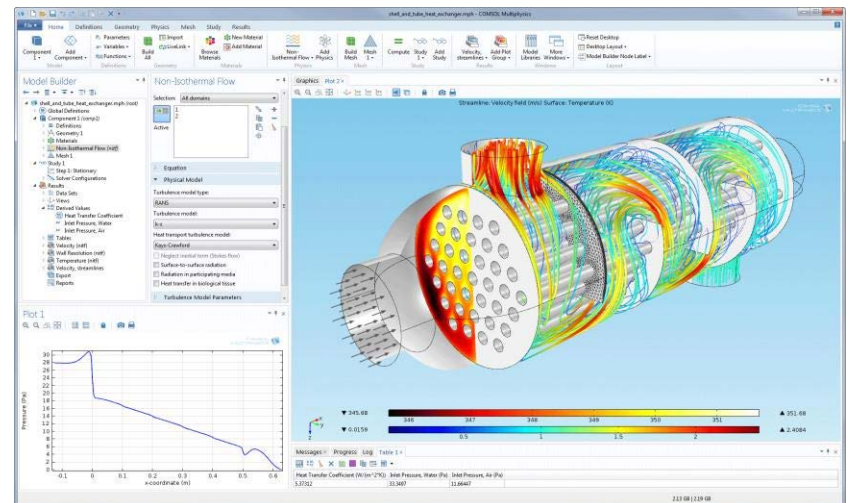
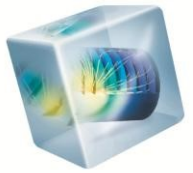
## **COMSOL Multiphysics**

**COMSOL Multiphysics** is a simulation environment designed with real-world applications in mind. The idea is to mimic as closely as possible effects that are observed in reality. To do this, there is the need to consider multiphysics.

## COMSOL allows to :

- Translate real-world physical laws into their virtual form
- Add any physical effect to your model
- Perform reliable simulations

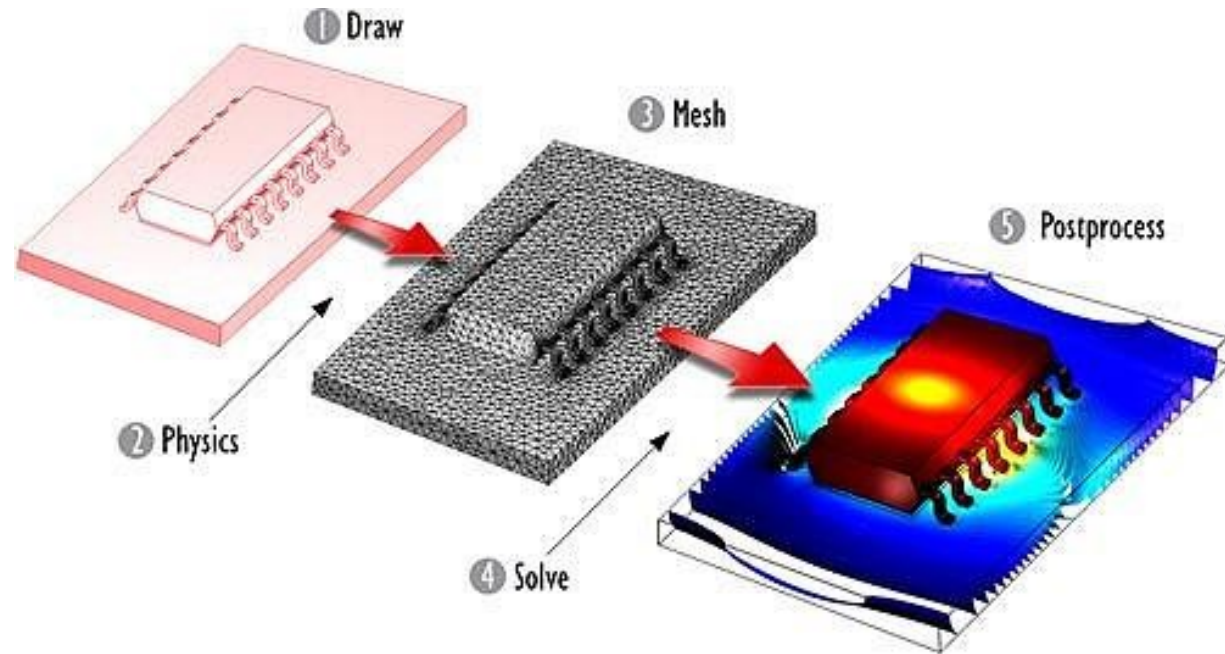
COMSOL  
MULTIPHYSICS®



# COMSOL Multiphysics

## Step Analysis Approach

- 1) Drawing **Geometry** and defining **Materials**
- 2) Adding **Physics**
- 3) Defining **Mesh**
- 4) Performing **Simulation**





## 5) Postprocessing of **Results**

# Content of the work

### ➤ Basic Example

**Structural Analysis of a Cantilever Beam with an applied distributed load**

- Analytical Solution using Beam Theory
- COMSOL Solution

### ➤ Thorough Example

**Multiphysics Simulation of a Busbar**

- Electrical Heating Analysis
- Structural Deformation Analysis

# Basic Example : Cantilever Beam

## Beam Theory Solution

### Data Set :

$$L = 3 \text{ m}$$

Length of the Beam

$$b = 0.1 \text{ m}$$

$$h = 0.1 \text{ m}$$

$$P = 1022.22 \text{ N/m}$$

$$E = 68.9 \text{ GPa}$$

$$\nu = 0.33$$

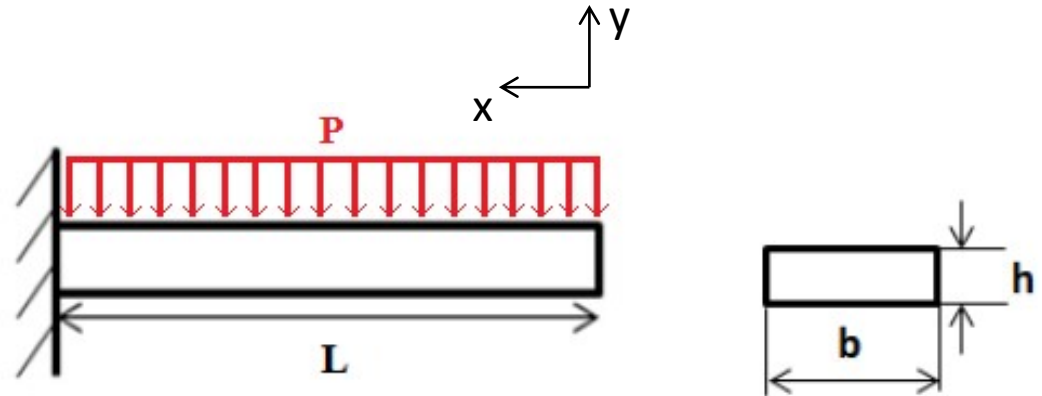
Cross Section Base

Cross Section Height

Distributed Load

Young's Modulus of Aluminum 6061-O

Poisson's Ratio



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$$\rho = 2700 \text{ kg/m}^3$$

$$Y = 55.2 \text{ MPa}$$

Density

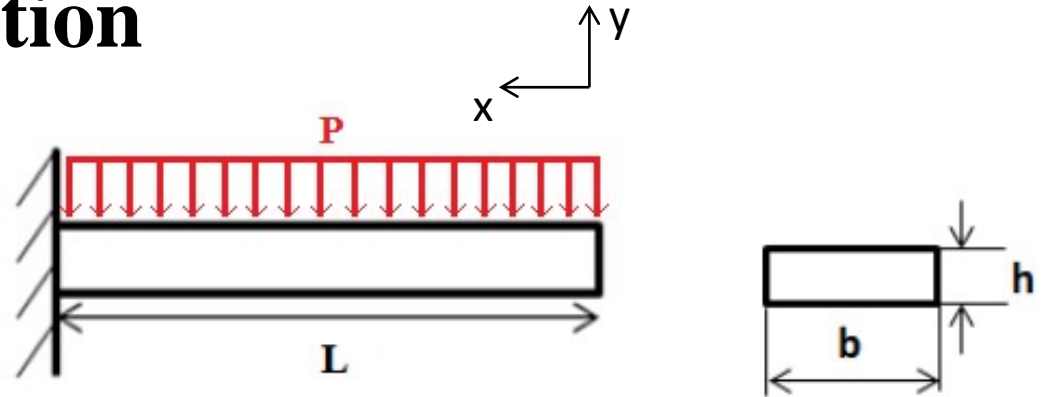
Yield Strength

# Basic Example : Cantilever Beam

## Beam Theory Solution

$$M(x) = \frac{P}{2} x^2$$

$$EI \frac{d^2 y}{dx^2} = \frac{P}{2} x^2$$



$$EI \int \int (x) = \int \left[ \frac{P}{2} x^2 dx \right] = \frac{P}{6} x^3 = C_1 \quad \text{at } x=L \quad \Rightarrow \quad \frac{P}{6} L^3 = 0, \quad C_1 = \frac{P}{6} L^3$$

$$P^3 dx = \frac{P}{6}$$

$$L^3 dx = \frac{24P}{6} x^4 = \frac{P}{6} L^3 x = C_2$$

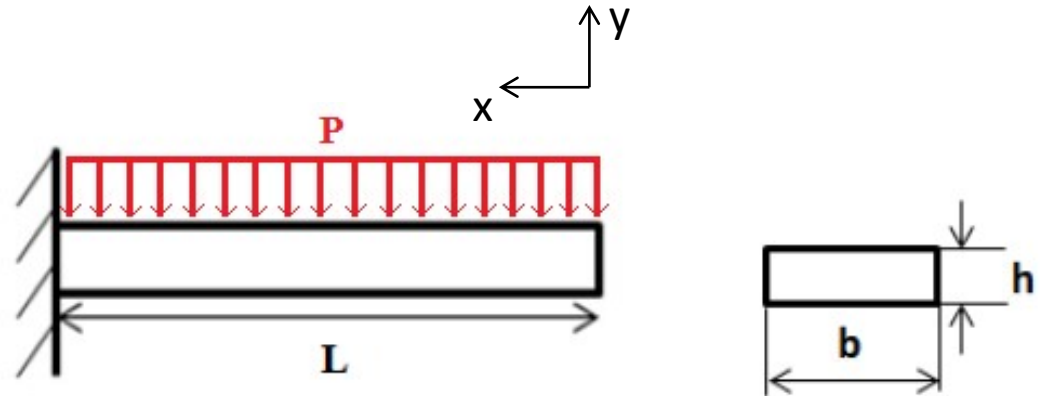


# Basic Example : Cantilever Beam

$$y(x) \square \frac{P}{24EI} x^4 \square \frac{P}{6EI} L^3 x \square \frac{P}{8EI} L^4$$

# Basic Example : Cantilever Beam

## Beam Theory Solution



$$y_{max} \square y(0) \square \frac{P}{8EI} L^4 \square 0.018026 \text{ m}$$

$$27.60 \text{ MPa} \square \frac{Mh}{2I} \square \frac{3P}{bh} \square \frac{1}{2} L^2 \square$$

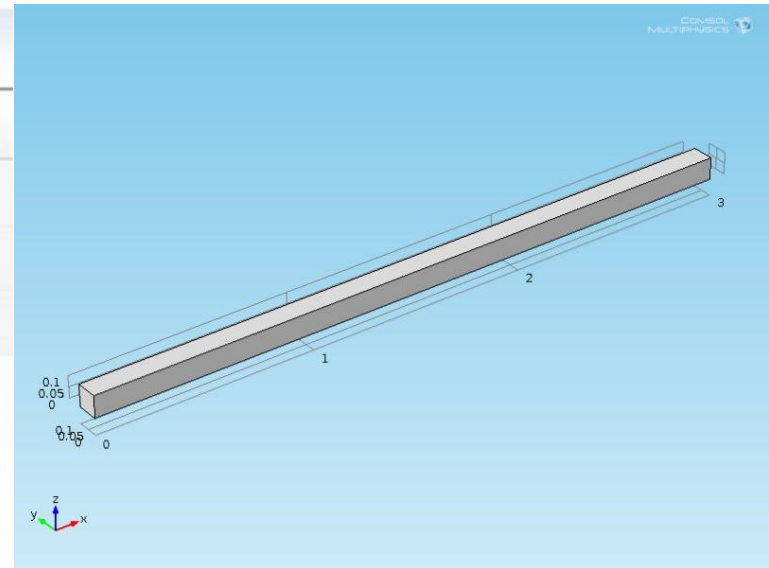
# Basic Example : Cantilever Beam

## COMSOL Solution

- **Geometry and Material Definition**

▼ Material Contents

	Property	Name	Value	Unit
✓	Young's modulus	E	689000...	Pa
✓	Poisson's ratio	nu	0.33	1
✓	Density	rho	2700	kg/...

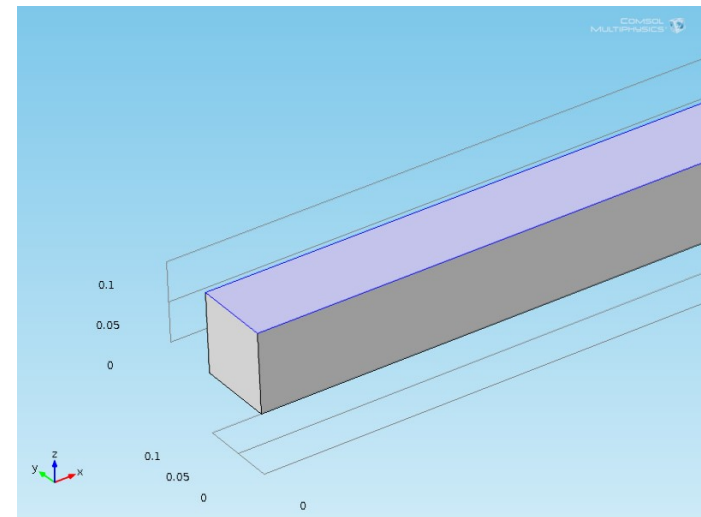
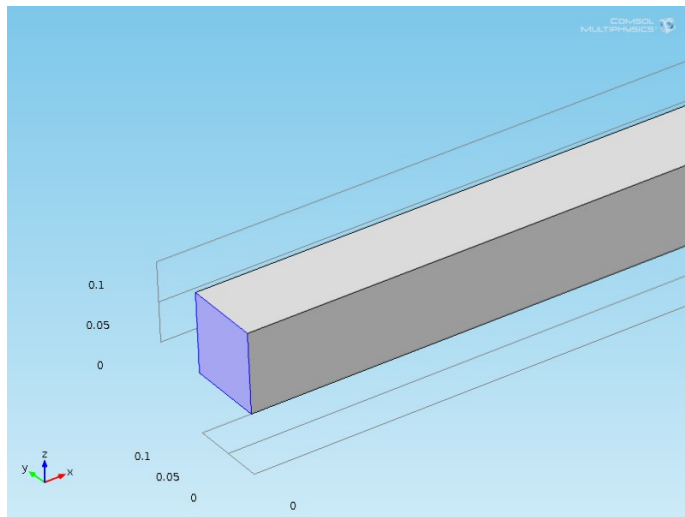


## COMSOL Solution



# Basic Example : Cantilever Beam

- **Physics & Boundary Conditions**  
“Solid Mechanics” module is selected and a fixed constraint together with a boundary load are defined



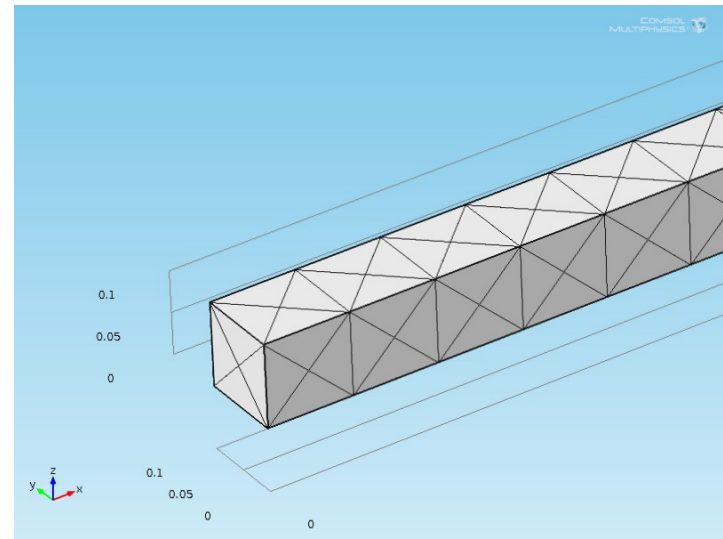
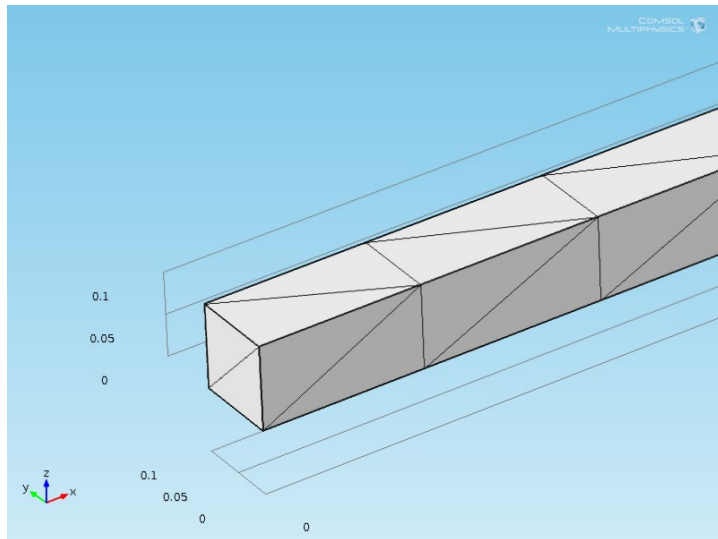
# Basic Example : Cantilever Beam

## COMSOL Solution

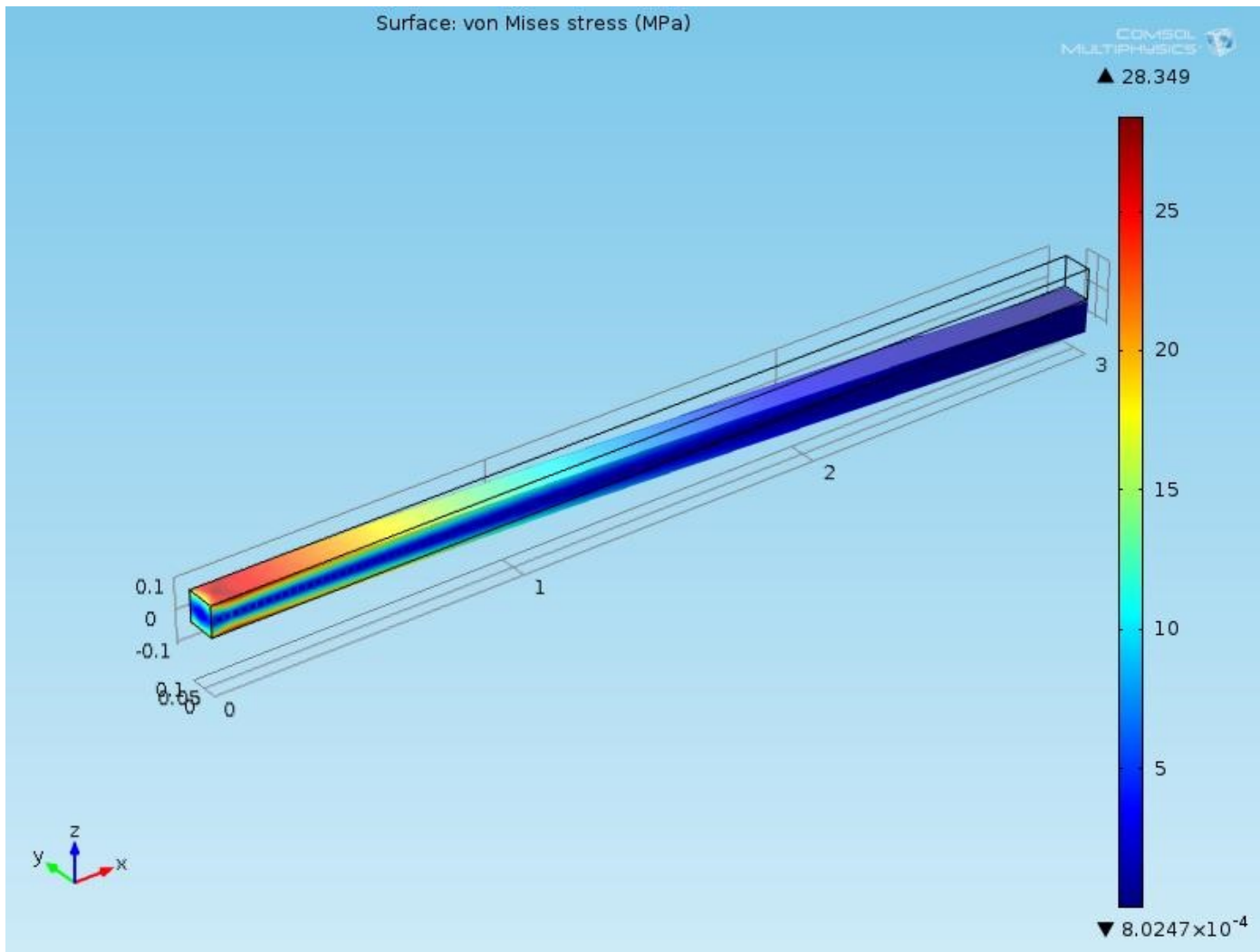
- **Mesh Definition**

A coarse mesh is firstly added to the model, then a finer mesh is chosen to perform a second simulation

# Basic Example : Cantilever Beam



# Basic Example : Cantilever Beam



# Basic Example : Cantilever Beam

## Beam Theory

$$y_{max} = 0.018026 \text{ m}$$

$$\sigma_{max} = 27.60 \text{ MPa}$$

## COMSOL Coarse

**Mesh**  $y_{max} =$   
 $0.017601 \text{ m}$   $\sigma_{max}$   
 $= 25.15 \text{ MPa}$

## COMSOL Fine

**Mesh**  $y_{max} =$   
 $0.017922 \text{ m}$   $\sigma_{max}$   
 $= 28.35 \text{ MPa}$

The percent error (%E) in our model max deflection/stress can be defined as:

$$\%E_{abs} = \frac{|x_{theoretical} - x_{model}|}{x_{theoretical}} \times 100$$

Coarse Mesh

$$\%E_y = 2.36\%$$

# Basic Example : Cantilever Beam

$$\%E_{\sigma} = 8.88$$

$\%$  **Fine Mesh**

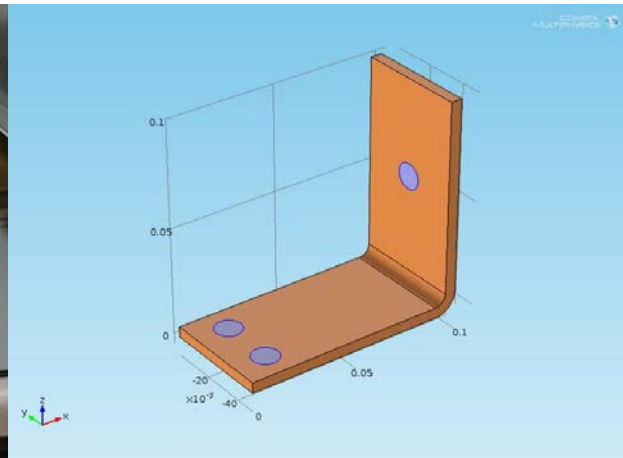
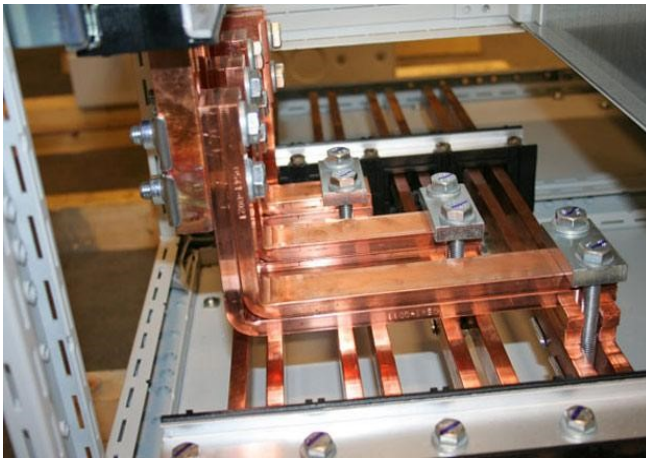
$$\%E_y = 0.58 \%$$

$$\%E_{\sigma} = 2.70 \%$$

# Thorough Example : Busbar

In electrical power distribution, a **busbar** is a strip or bar of copper, brass or aluminum that conducts electricity within an electrical apparatus.

The cross-sectional size of the busbar determines the maximum amount of current that can be safely carried.



# Busbar : Joule Heating

The current conducted in the busbar, from the top bolt to the bolts on the bottom, produces heat due to resistive losses, a phenomenon referred to as **Joule Heating**.

- **Geometry and Material Definition**



# Busbar : Joule Heating

## Geometry

Parameters			
Name	Expression	Value	Description
L	9[cm]	0.090000 m	length of busbar
rad_1	6[mm]	0.0060000 m	radius of bolts
tbb	5[mm]	0.0050000 m	thickness of busbar
wbb	5[cm]	0.050000 m	width of busbar

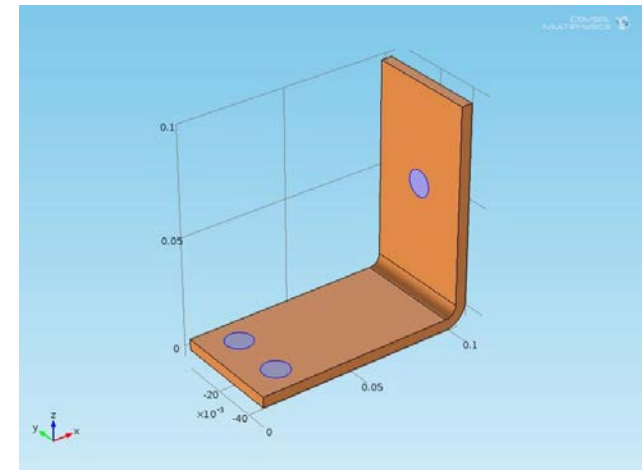
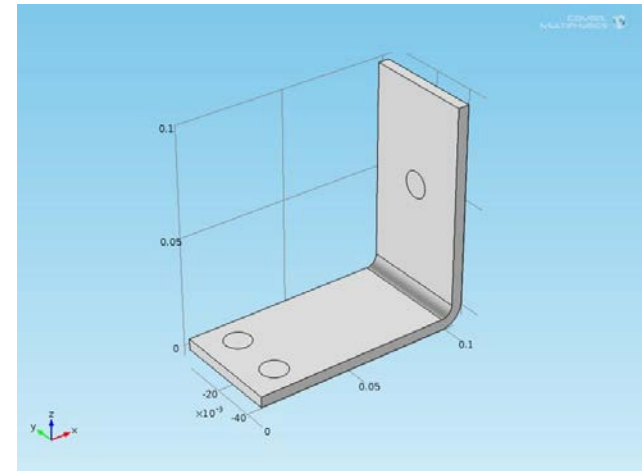
## Materials

Busbar Material : **Copper**

Electrical Conductivity :  $\sigma_{Cu} = 5.998 \cdot 10^7$  S/m

Bolts Material : **Titanium beta-21S**

Electrical Conductivity :  $\sigma_{Ti} = 7.407 \cdot 10^5$  S/m



# Busbar : Joule Heating

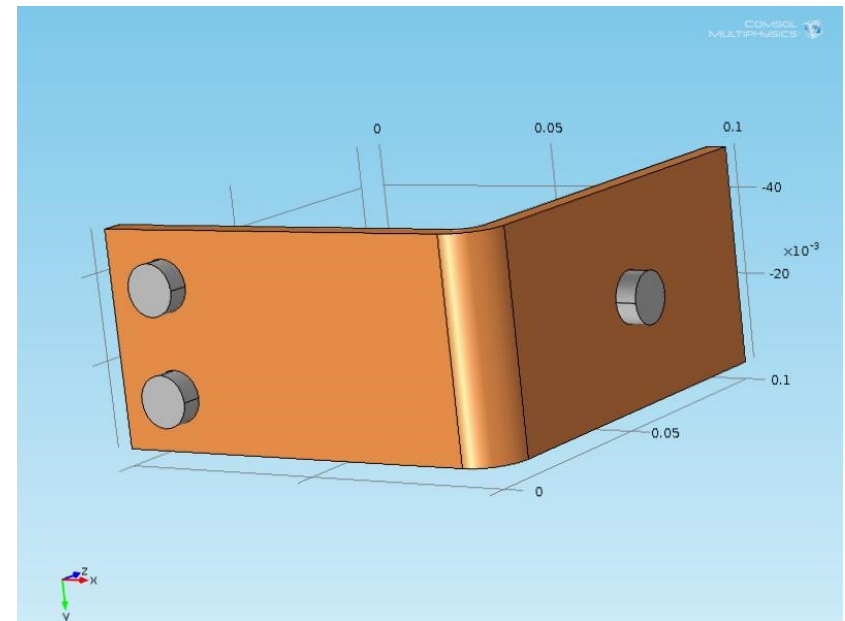
- **Physics & Boundary Conditions**

“Joule Heating” module is selected and the boundary conditions for the heat transfer problem and the conduction of electric current are set as follows. **Boundary**      **Conditions**

**Heat Transfer** : Circular bolt

boundaries neither heated are  
nor assumed cooled to by be the  
surroundings.

**Electric Current** : The single  
Titanium bolt is set to an electric  
potential of 20 mV , whereas the

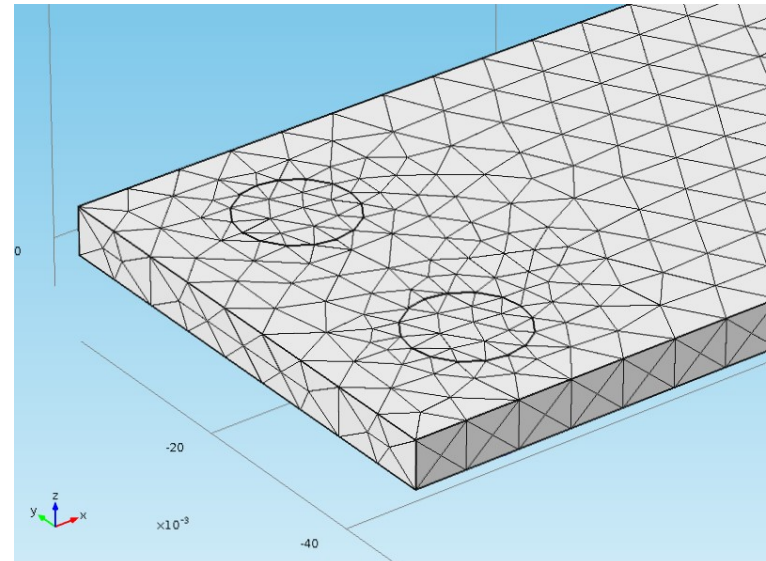
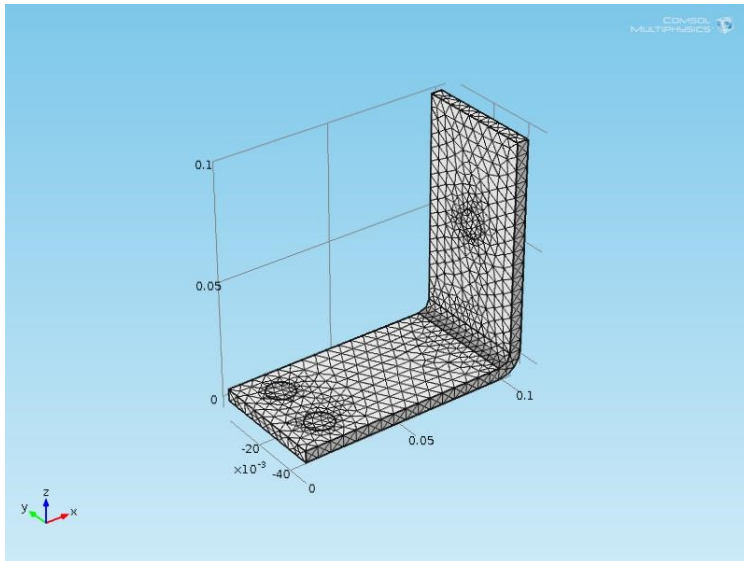


# Busbar : Joule Heating

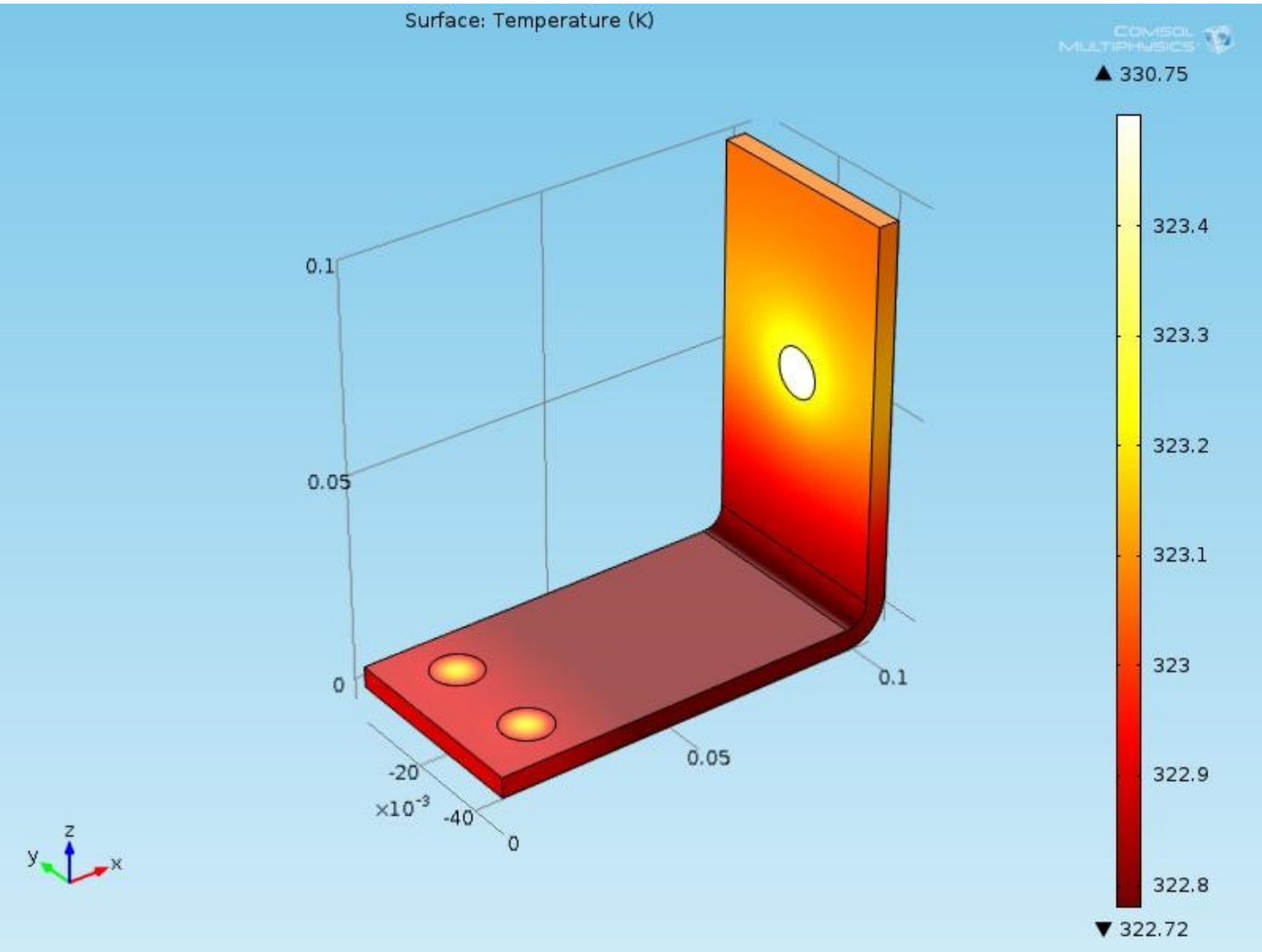
two remaining bolts are set to ground.

- **Mesh Definition**

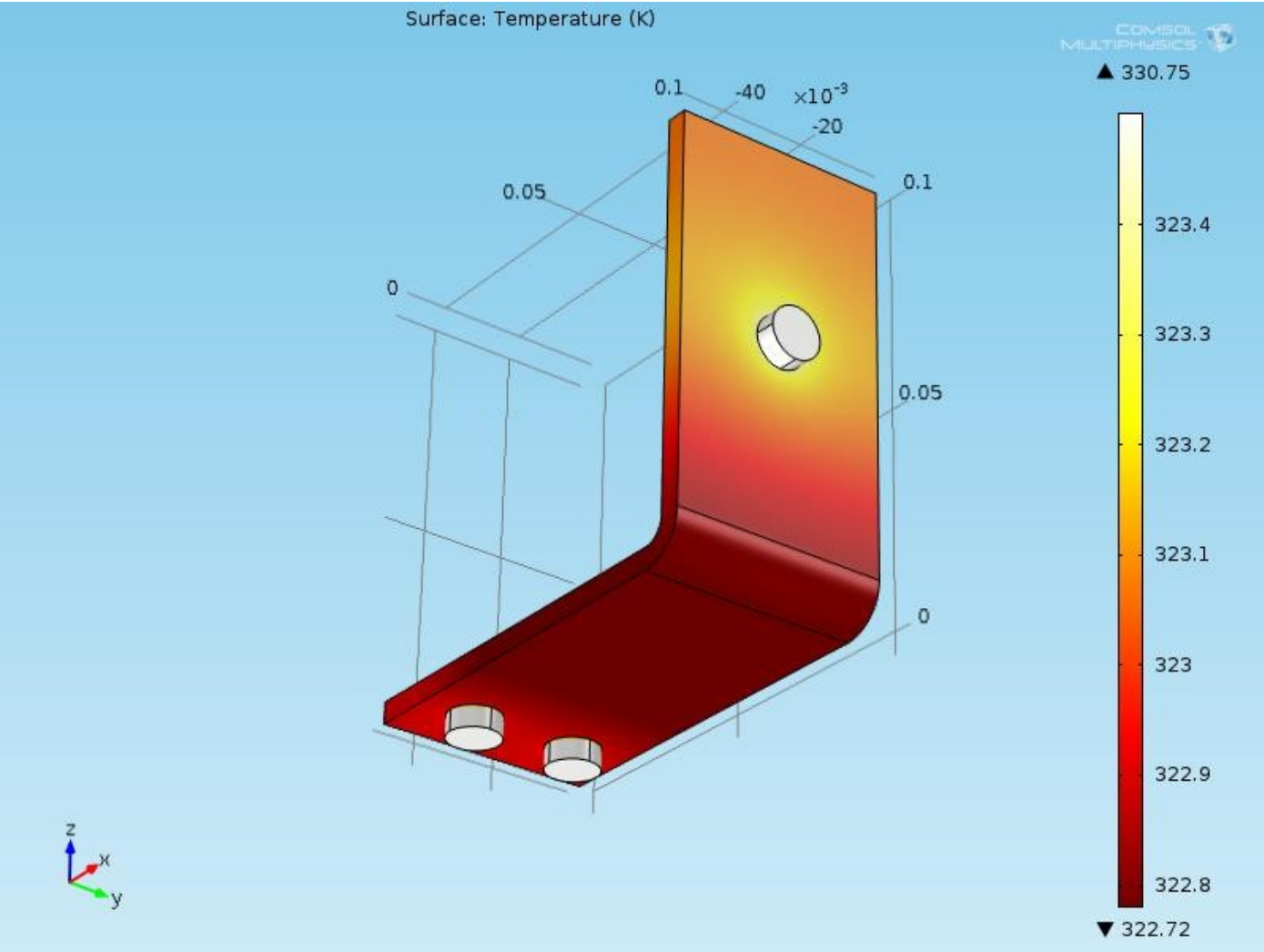
A physics-controlled mesh is created by default. It is important to notice that the number of elements on curved boundaries is higher.



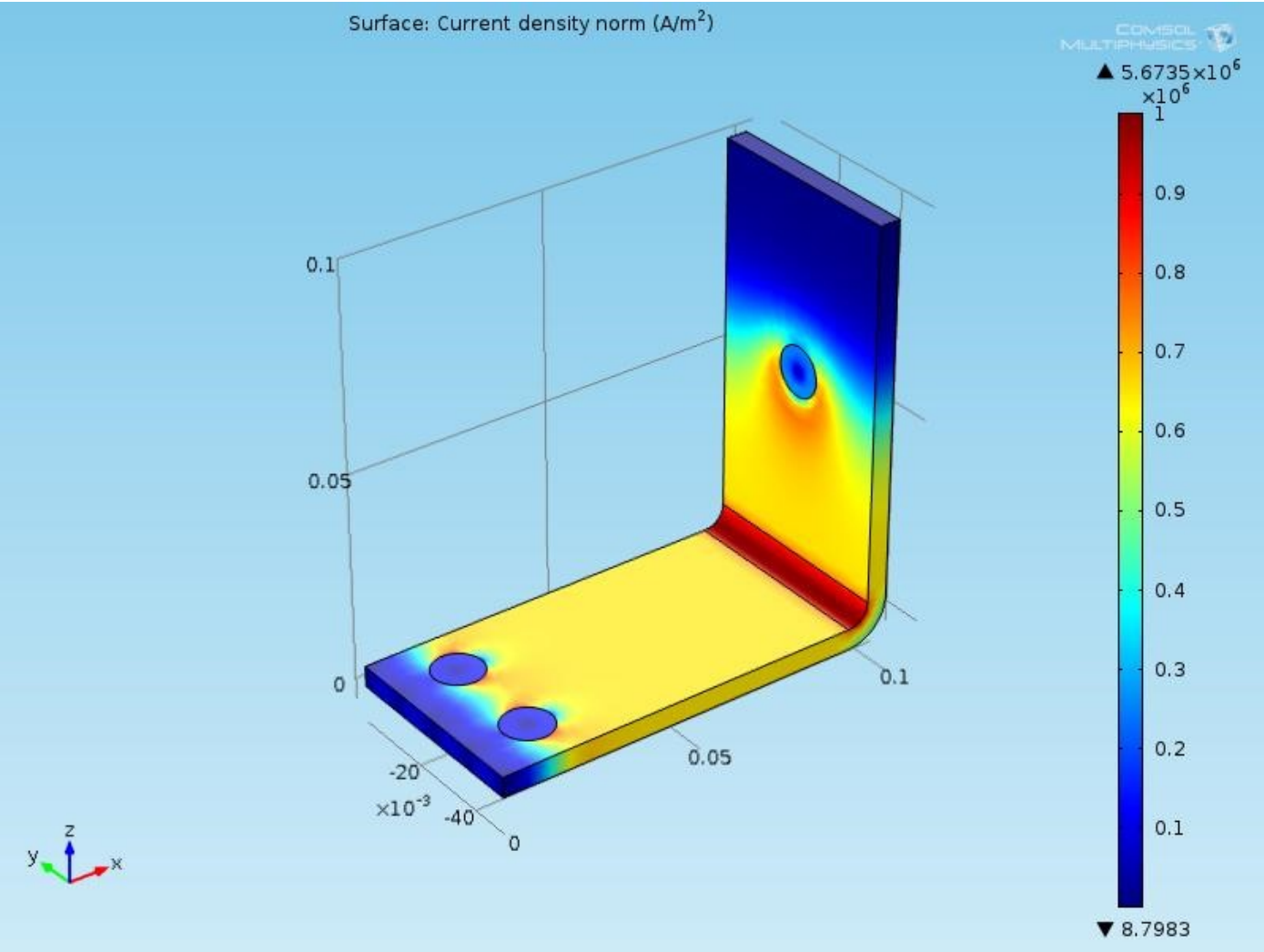
# Busbar : Joule Heating



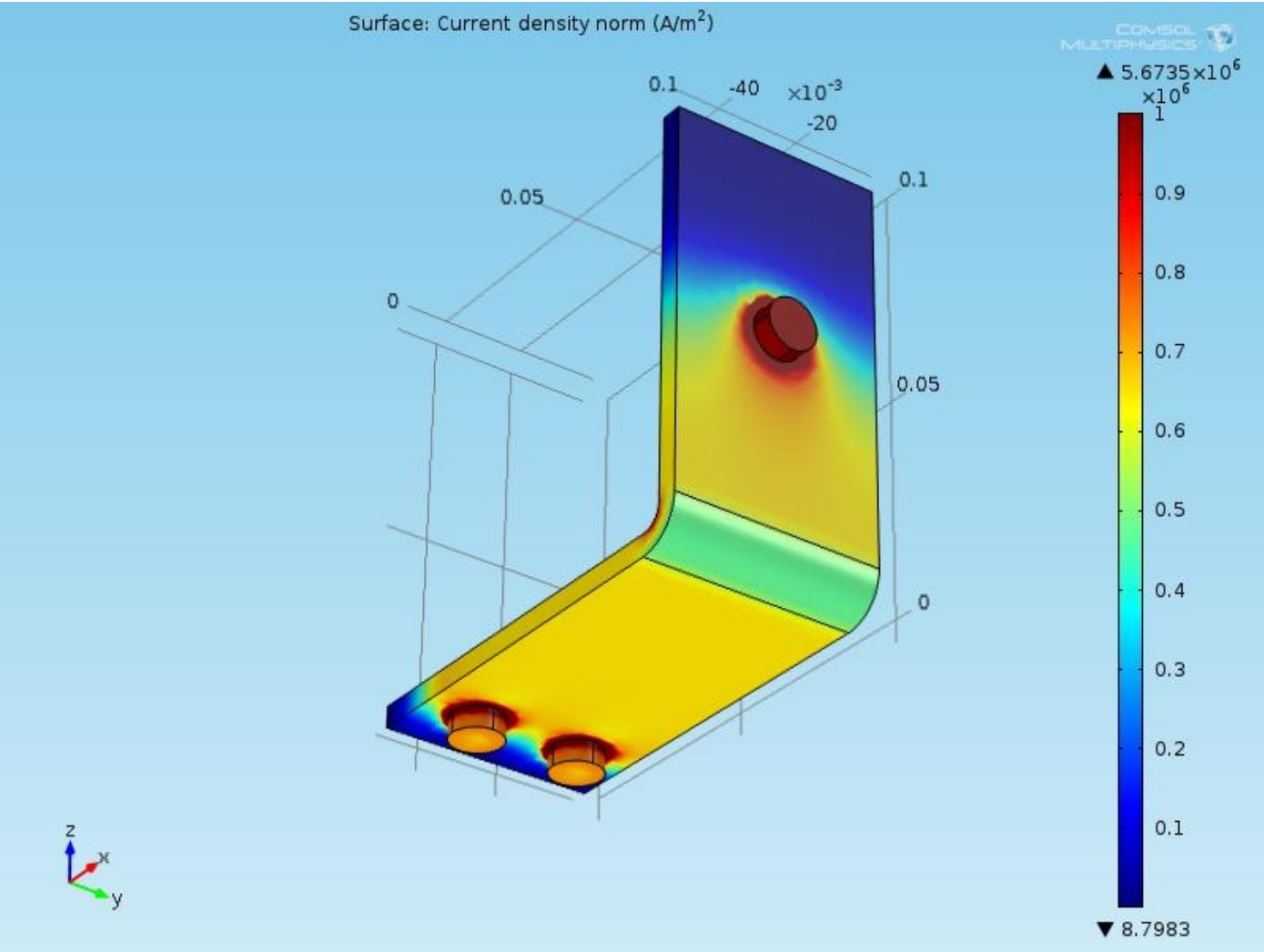
# Busbar : Joule Heating



# Busbar : Joule Heating



# Busbar : Joule Heating



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# Busbar : Joule Heating



# Adding Physics to the Model

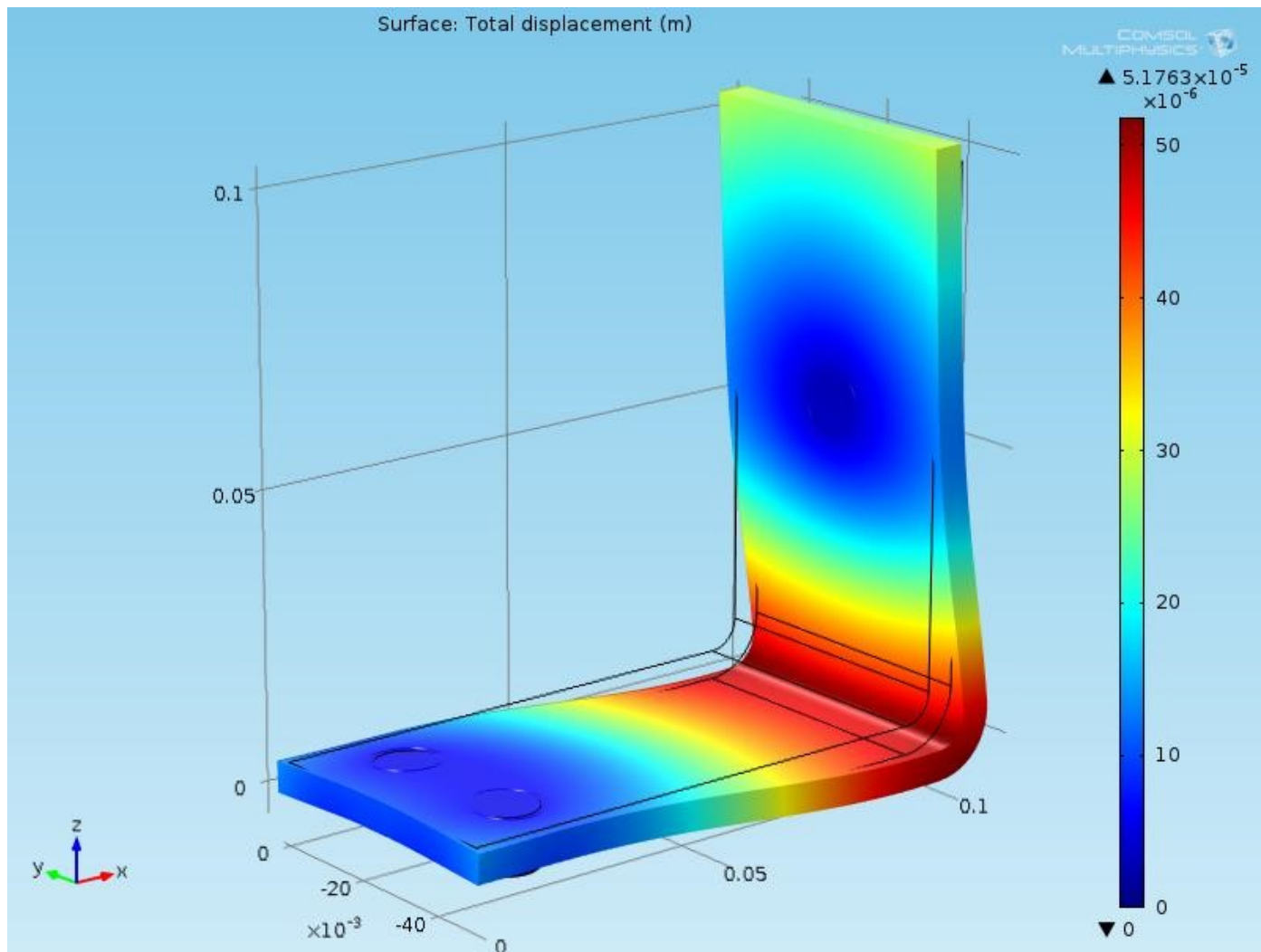
The Joule Heating simulation has just shown that there is a temperature rise in the busbar. So it is logical to ask :

**What kind of mechanical stress is induced by thermal expansion?**

To answer this question we can **simply expand the existing model** to include the physics associated with **“Structural Mechanics”**.

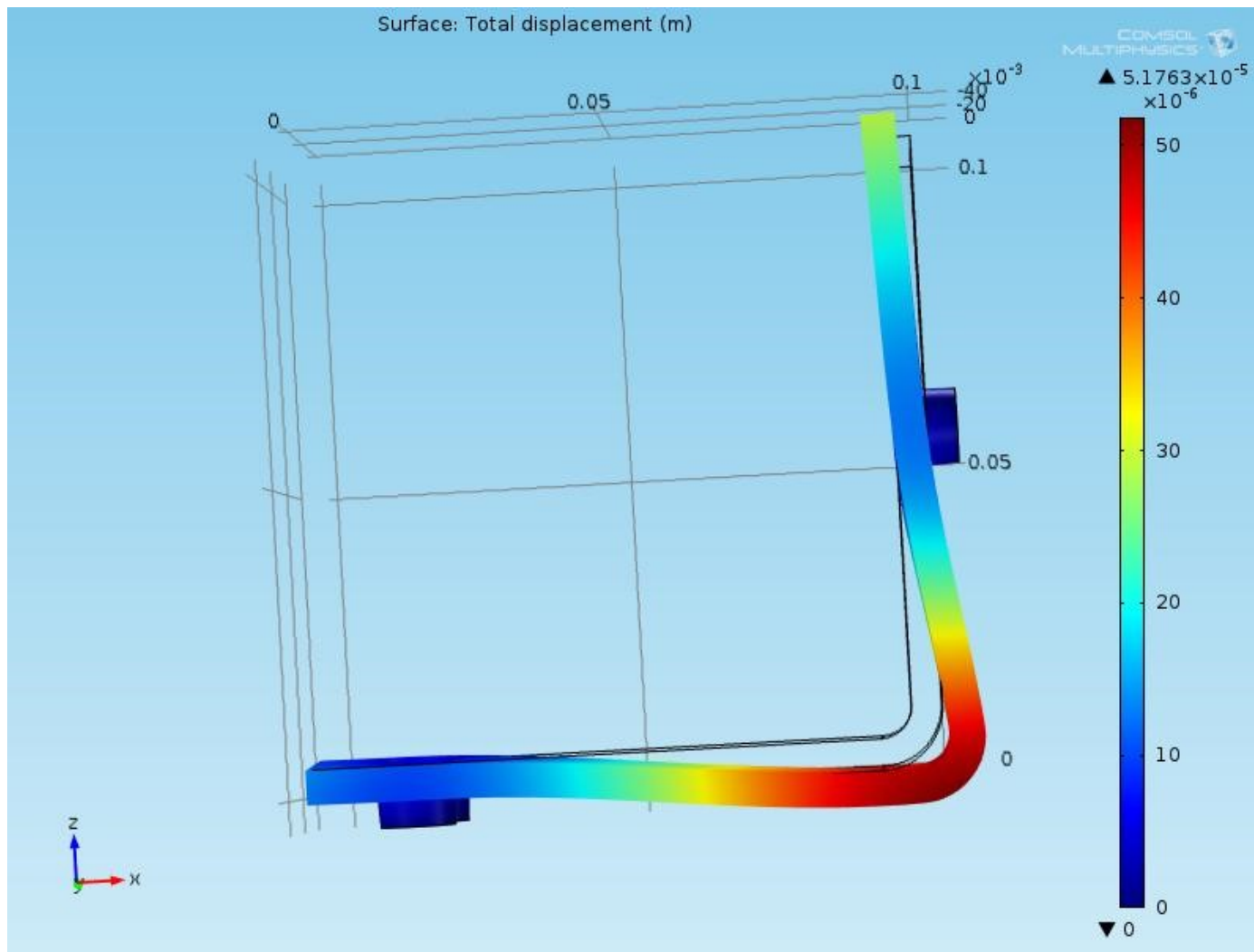
The Joule Heating module and the Structural Mechanics module are then coupled to perform an overall simulation which includes both the effects. **We can now perform a true Multiphysics Simulation!**

# Busbar : Structural Deformation



- **Results : Total Displacement**

# Busbar : Structural Deformation

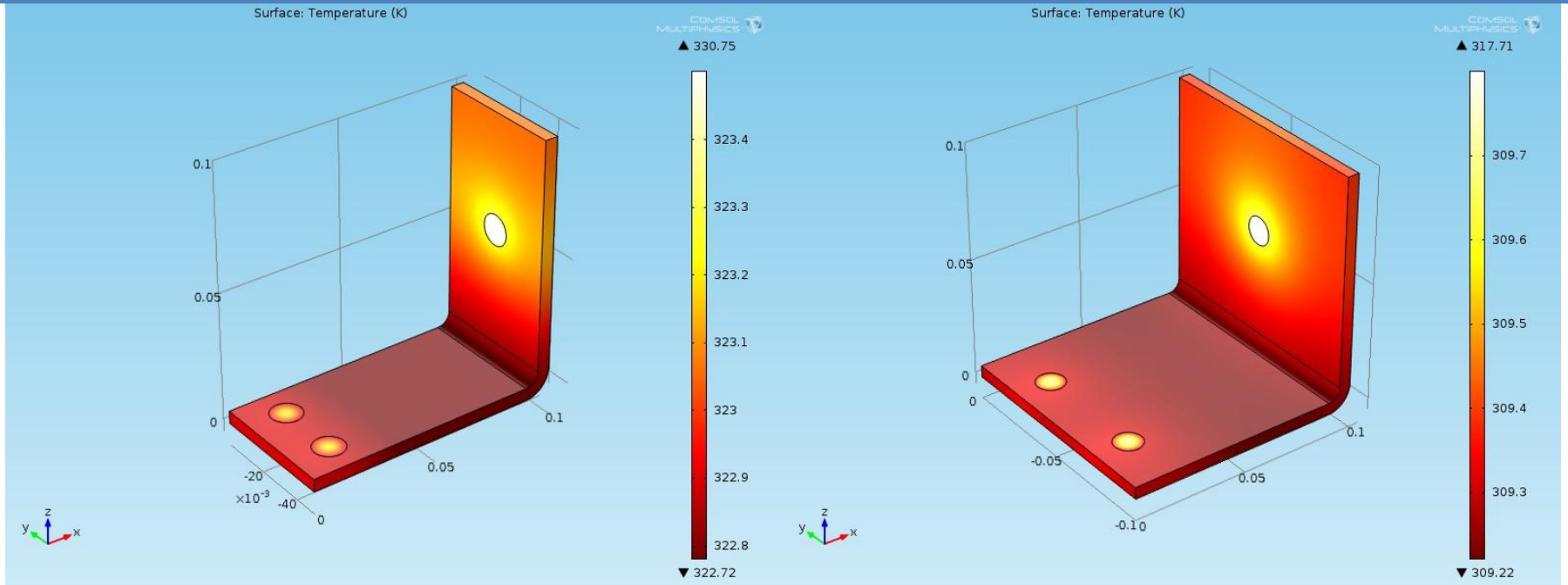


- **Results : Total Displacement**

# Sweeping a Geometric Parameter

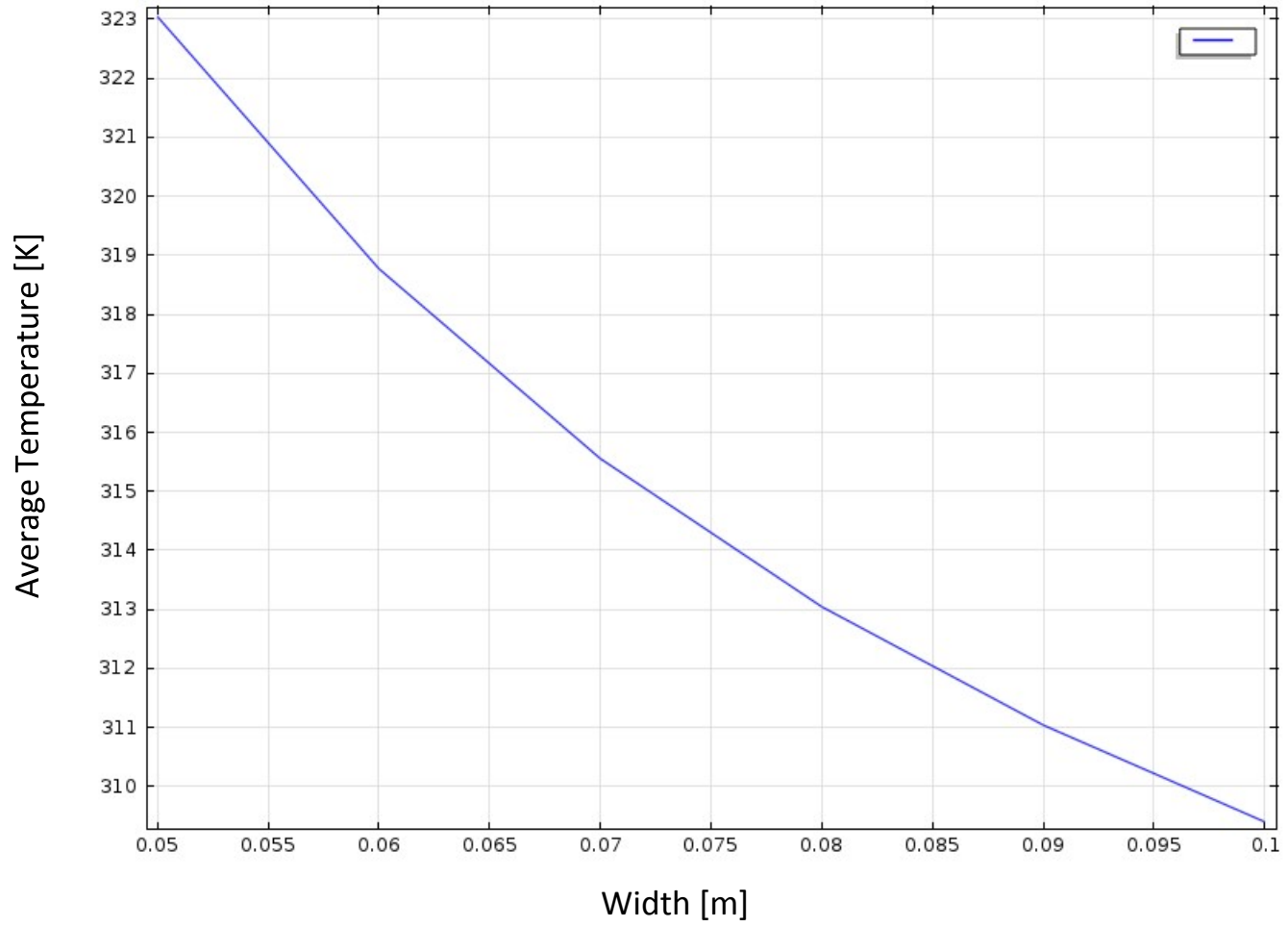
Often it is interesting to generate multiple instances of a design to meet specific constraints. For the busbar, a design goal might be to lower the operating temperature by varying its geometry.

- Initial busbar width = 5 cm ,  $T_{\max} = 330$  K
- New busbar width = 10 cm ,  $T_{\max} = 317$  K



# Sweeping a Geometric Parameter

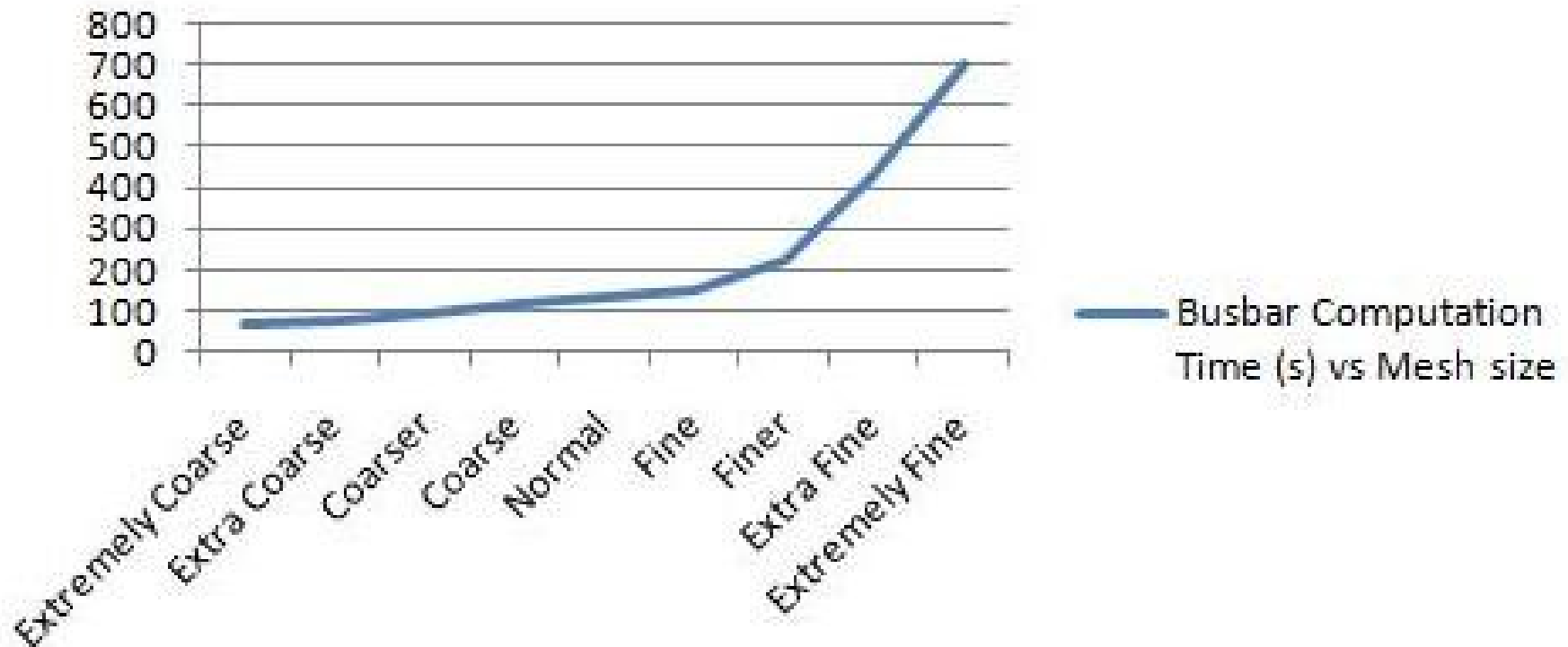
The plot shows that the average temperature decreases as the width increases.





# Busbar Computation Time vs Mesh Size

The computational time decreases with the coarseness of the mesh size, as shown by the plot.



# Conclusions

- ✓ The Cantilever Beam analysis shows that computer simulation can provide reliable results (as predicted in theory)
- ✓ It is possible to add several physical domains without the need of creating a disconnected model each time
- ✓ It is possible to test various options of a product more quickly and efficiently
- ✓ It is important to find a common balance between

computation speed and precision of results  
(“Parallel Computing” for very time consuming simulations)

*Thank you for  
your attention!*